CHAPTER 4
POWER MEASUREMENTS

4.1 INTRODUCTION

Electric power is the rate of doing work. It is expressed in Watts. The higher units of power used in practice include kilowatts, megawatts, etc. \( P_{\text{Watts}} = VI \cos \phi \), i.e., a power of one watt is said to be expended when a source of one volt passes a current of one ampere through a load resistance/impedance of one ohm at unity power factor.

The power measurements are made with the help of a wattmeter. Wattmeter is an indicating deflecting type of instrument used in laboratories for measurement of power in various ranges. A wattmeter consists of two coils as shown in the schematic representative figure 4.1.

- **Current coil (CC):** connected in series with circuit and carries the load current. It is designed such that it is wound with 2 to 3 turns of thick wire and hence it has a very low resistance.
- **Voltage or Pressure or Potential coil (PC):** connected across the load circuit and hence carries a current proportional to the load current. The total load voltage appears across the PC. It is designed such that it is wound with several turns of thin wire and hence it has a very high resistance.

The wattmeter can be a UPF meter or LPF meter depending on the type of the load connected in the measuring circuit. For power measurements in AC circuits, the wattmeter is widely adopted. In principle and construction, it is a combination of those applicable for an ammeter and a voltmeter.

The electrical power can be of three forms:

- **Real power** or simply, the power is the power consumed by the resistive loads on the system. It is expressed in watts (W). This is also referred as true power, absolute power, average power, or wattage.
- **Reactive power** is the power consumed by the reactive loads on the system. It is expressed in reactive volt-amperes (VAR).
- **Apparent power** is the vector sum of the above two power components. It is expressed in volt-amperes (VA).

Thus, it is observed from the power triangle shown in figure 4.2, that more is the deviation of power factor from its unity value, more is the deviation of real power from the apparent power. Also, we have
\[ VA^2 = W^2 + VAr^2 \]  
And power factor,  \( \cos \varphi = \frac{\text{Watts}}{\text{VA}} \)  

4.2 SINGLE PHASE REAL POWER MEASUREMENTS

4.2.1 Electrodynamometer Wattmeter

An electrodynamometer wattmeter consists of two fixed coils, \( F_A \) and \( F_B \) and a moving coil \( M \) as shown in figure 4.3. The fixed coils are connected in series with the load and hence carry the load current. These fixed coils form the current coil of the wattmeter. The moving coil is connected across the load and hence carries a current proportional to the voltage across the load. A highly non-inductive resistance \( R \) is put in series with the moving coil to limit the current to a small value. The moving coil forms the potential coil of the wattmeter.

Fig. 4.3 Electrodynamometer Wattmeter

The fixed coils are wound with heavy wire of minimum number of turns. The fixed coils embrace the moving coil. Spring control is used for movement and damping is by air. The deflecting torque is proportional to the product of the currents in the two coils. These watt meters can be used for both DC and AC measurements. Since the deflection is
proportional to the average power and the spring control torque is proportional to the deflection, the scale is uniform. The meter is free from waveform errors. However, they are more expensive.

**Expression for the deflection torque:**
Let \( i_C, i_P \): Current in the fixed and moving coils respectively,
- \( M \): Mutual inductance between the two coils,
- \( \theta \): Steady final deflection of the instrument,
- \( K \): Spring constant,
- \( V, I \): RMS values of voltage and current in the measuring circuit and
- \( R_P \): Pressure coil resistance.

Instantaneous voltage across pressure coil, \( v = \sqrt{2} V \sin wt \)

Instantaneous current in the pressure coil, \( i_P = \sqrt{2} \frac{V}{R_P} \sin wt = \sqrt{2} I_P \sin wt \)

Instantaneous current in the current coil, \( i_C = \sqrt{2} I \sin (wt-\phi) \)

Instantaneous torque is given by: \( \mathbf{T}_i = i_C i_P \left( \frac{dM}{d\theta} \right) \)

\[
\mathbf{T}_i = \left[ \sqrt{2} I \sin (wt-\phi) \right] \left[ \sqrt{2} I_P \sin wt \right] \frac{dM}{d\theta} \quad (4.3)
\]

Average deflecting torque, \( \mathbf{T}_d = \frac{1}{T} \int_0^T \mathbf{T}_i \, dwt \)

\[
\mathbf{T}_d = \frac{1}{T} \int_0^T I_P I \left[ \cos \phi - \cos (2wt - \phi) \right] \left( \frac{dM}{d\theta} \right) \, dwt
\]

\[
= \left( \frac{V I}{R_P} \right) \cos \phi \left( \frac{dM}{d\theta} \right) \quad (4.4)
\]

Since the controlling torque, \( \mathbf{T}_c = K\theta \), we have at balance of the moving pointer, \( \mathbf{T}_d = \mathbf{T}_c \), so that,

\[
\theta = \left[ \frac{V I \cos \phi}{(K R_P)} \right] \left( \frac{dM}{d\theta} \right) \quad (4.5)
\]

Where \( K' = K R_P \) and \( P \) is the power consumption. Thus the deflection of the wattmeter is found to be the direct indication of the power being consumed in the load circuit.

### 4.2.2 Low Power Factor Wattmeter

If an ordinary electrodynamometer wattmeter is used for measurement of power in low power factor circuits, (PF<0.5), then the measurements would be difficult and inaccurate since:

- The deflecting torque exerted on the moving system will be very small and
- Errors are introduced due to pressure coil inductance (which is large at LPF)

Thus, in a LPF wattmeter, special features are incorporated in a general electrodynamometer wattmeter circuit to make it suitable for use in LPF circuits as under:

(a) **Pressure coil current:**

The pressure coil circuit is designed to have a low value of resistance so that the current through the pressure coil is increased to provide an increased operating torque.
(b) Compensation for pressure coil current:
On account of low power factor, the power is small and the current is high. In this context, there are two possible connections of the potential coil of a wattmeter as shown in figure 4.4. The connection (a) can not be used, since owing to the high load current, there would be a high power loss in the current coil and hence the wattmeter reading would be with a large error. If the connection (b) is used, then the power loss in the pressure coil circuit is also included in the meter readings.

Thus it is necessary to compensate for the pressure coil current in a low power factor wattmeter. For this, a compensating coil is used in the instrument to compensate for the power loss in the pressure coil circuit as shown in figure 4.5.

(c) Compensation for pressure coil inductance:
At low power factor, the error caused by the pressure coil inductance is very large. Hence, this has to be compensated, by connecting a capacitor C across a portion of the series resistance in the pressure coil circuit as shown in figure 4.5.

(d) Realizing a small control torque:
Low power factor wattmeters are designed to have a very small control torque so that they can provide full scale deflection (f.s.d.) for power factor values as low as 10%. Thus, the complete circuit of a low power factor wattmeter is as shown in figure 4.5.
4.3 REACTIVE POWER MEASUREMENTS

A single wattmeter can also be used for three phase reactive power measurements. For example, the connection of a single wattmeter for 3-phase reactive power measurement in a balanced three phase circuit is as shown in figure 4.6.

![Fig. 4.6 Reactive power measurement circuit](image)

The current coil of the wattmeter is inserted in one line and the potential coil is connected across the other two lines. Thus, the voltage applied to the voltage coil is \( V_{RB} = V_R - V_B \), where, \( V_R \) and \( V_B \) are the phase voltage values of lines R and B respectively, as illustrated by the phasor diagram of figure 4.7.

![Fig. 4.7 Phasor diagram for reactive power measurements](image)

The reading of the wattmeter, \( W_{3ph} \) for the connection shown in figure 4.6 can be obtained based on the phasor diagram of figure 4.7, as follows:

Wattmeter reading,
\[
W_{ph} = I_y V_{RB} = I_y V_L \cos (90 + \phi) = - \sqrt{3} V_{ph} I_{ph} \sin \phi = - \sqrt{3} \text{ (Reactive power per phase)} \quad (4.6)
\]
Thus, the three phase power, $W_{3\text{ph}}$, is given by,

$$W_{3\text{ph}} = 3 \ (\text{VARs/phase})$$

$$= 3 [W_{\text{ph}} / \sqrt{3}]$$

$$= - \sqrt{3} \ (\text{wattmeter reading}) \quad (4.7)$$

### 4.4 THREE PHASE REAL POWER MEASUREMENTS

The three phase real power is given by,

$$P_{3\text{ph}} = 3 V_{\text{ph}} I_{\text{ph}} \cos \theta$$

or

$$P_{3\text{ph}} = \sqrt{3} V_L I_L \cos \theta \quad (4.8)$$

The three phase power can be measured by using either one wattmeter, two wattmeters or three wattmeters in the measuring circuit. Of these, the two wattmeter method is widely used for the obvious advantages of measurements involved in it as discussed below.

#### 4.4.1 Single Wattmeter Method

Here only one wattmeter is used for measurement of three phase power. For circuits with the balanced loads, we have: $W_{3\text{ph}} = 3 (\text{wattmeter reading})$. For circuits with the unbalanced loads, we have: $W_{3\text{ph}} = \text{sum of the three readings obtained separately by connecting wattmeter in each of the three phases}$. If the neutral point is not available (3 phase 3 wire circuits) then an artificial neutral is created for wattmeter connection purposes. Instead three wattmeters can be connected simultaneously to measure the three phase power. However, this involves more number of meters to be used for measurements and hence is not preferred in practice. Instead, the three phase power can be easily measured by using only two wattmeters, as discussed next.

#### 4.4.2 Two Wattmeter Method

The circuit diagram for two wattmeter method of measurement of three phase real power is as shown in the figure 4.7. The current coil of the wattmeters $W_1$ and $W_2$ are inserted respectively in $R$ and $Y$ phases. The potential coils of the two wattmeters are joined together to phase $B$, the third phase. Thus, the voltage applied to the voltage coil of the meter, $W_1$ is $V_{RB} = V_R - V_B$, while the voltage applied to the voltage coil of the meter, $W_2$ is $V_{YB} = V_Y - V_B$, where, $V_R$, $V_B$ and $V_C$ are the phase voltage values of lines $R$, $Y$ and $B$ respectively, as illustrated by the phasor diagram of figure 4.8. Thus, the reading of the two wattmeters can be obtained based on the phasor diagram of figure 4.8, as follows:

$$W_1 = I_R V_{RB}$$

$$= I_L V_L \cos (30 - \theta) \quad (4.9)$$

$$W_2 = I_Y V_{YB}$$

$$= I_L V_L \cos (30 + \theta) \quad (4.10)$$

Hence,

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \theta = P_{3\text{ph}} \quad (4.11)$$

And

$$W_1 - W_2 = V_L I_L \sin \theta \quad (4.12)$$

So that then,

$$\tan \theta = \sqrt{3} [W_1 - W_2] / [W_1 + W_2] \quad (4.13)$$

Where $\theta$ is the lagging PF angle of the load. It is to be noted that the equations (4.11) and (4.12) get exchanged if the load is considered to be of leading PF.
The readings of the two wattmeters used for real power measurements in three phase circuits as above vary with the load power factor as described in the table 4.1.

**Table 4.1 Variation of wattmeter readings with load PF (lag)**

<table>
<thead>
<tr>
<th>PF angle (lag)</th>
<th>PF</th>
<th>$W_1$</th>
<th>$W_2$</th>
<th>$W_{3ph}=W_1+W_2$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ (lag)</td>
<td>cos $\phi$</td>
<td>$V_L I_L \cos(30-\phi)$</td>
<td>$V_L I_L \cos(30+\phi)$</td>
<td>$\sqrt{3}V_L I_L \cos\phi$</td>
<td>Gen. Case (always $W_1 \geq W_2$)</td>
</tr>
<tr>
<td>$0^0$</td>
<td>UPF</td>
<td>$\sqrt{3}/2 \ V_L I_L$</td>
<td>$\sqrt{3}/2 \ V_L I_L$</td>
<td>$2W_1$ or $2W_2$</td>
<td>$W_1=W_2$</td>
</tr>
<tr>
<td>$30^0$</td>
<td>0.866</td>
<td>$V_L I_L$</td>
<td>$V_L I_L /2$</td>
<td>$1.5W_1$ or $3W_2$</td>
<td>$W_2=W_1/2$</td>
</tr>
<tr>
<td>$60^0$</td>
<td>0.5</td>
<td>$\sqrt{3}/2 \ V_L I_L$</td>
<td>ZERO</td>
<td>$W_1$ alone</td>
<td>$W_2$ reads zero</td>
</tr>
<tr>
<td>$&gt;60^0$</td>
<td>&lt;0.5</td>
<td>$W_1$</td>
<td>$W_2$ reads negative</td>
<td>$W_1+(-W_2)$</td>
<td>For taking readings, the PC or CC connection of $W_2$ should be reversed (LPF case)</td>
</tr>
</tbody>
</table>
4.5 SOLVED PROBLEMS

1. A 3-phase, 10 kVA load has a PF of 0.342. The power is measured by two wattmeter method. Find the reading of each wattmeter when the PF is (i) Lagging and (ii) Leading.

**Solution:**

The total VA, \( S_{3\text{ph}} = \sqrt{3}V_LI_L \)

i.e., \( 10 \times 10^3 = \sqrt{3}V_LI_L \)

Thus, \( V_LI_L = 5.7735 \text{ kVA} \)

(i) Lagging PF:

\( W_1 = V_LI_L \cos(30-\phi) = 5.7735 \times 10^3 \cos(30-70) = 4.4228 \text{ kW} \) and

\( W_2 = V_LI_L \cos(30+\phi) = 5.7735 \times 10^3 \cos(30+70) = -1.0026 \text{ kW} \)

(ii) Leading PF:

\( W_1 = V_LI_L \cos(30+\phi) = -1.0026 \text{ kW} \) and \( W_2 = V_LI_L \cos(30-\phi) = 4.4228 \text{ kW} \)

**Note:** It can be observed that when the PF is changed from lagging to leading, the readings of wattmeters \( W_1 \) and \( W_2 \) get interchanged.

2. A 3-phase, 400 V load has a PF of 0.6 lagging. The two wattmeters read a total input power of 20 kW. Find reading of each wattmeter.

**Solution:**

\( W_1 + W_2 = P_{3\text{ph}} = 20000 \text{ W}, \ V_L = 400 \text{ V}, \ \cos \phi = 0.6 \)

i.e., \( 20 \times 10^3 = \sqrt{3} (400) I_L (0.6); \) solving, \( I_L = 48.125 \text{ A}, \ \phi = 53.13^0 \)

\( W_1 = V_LI_L \cos(30-\phi) = 17.698 \text{ kW} \)

\( W_1 = V_LI_L \cos(30+\phi) = 2.302 \text{ kW} \)

4.6 EXERCISES

1. The two wattmeter method is used to measure power consumed by a delta connected load. Each branch of load has an impedance of \( 20 \angle 60^0 \). Supply voltage is 400 V. calculate the total power and the readings on the individual wattmeters.

2. The power input measurement to a synchronous motor is done using two wattmeter method. Each of the wattmeter reads 40 kW at a certain operating condition. If now, the PF is changed to 0.8 lagging, what would be the new wattmeter readings?

   (Ans. : 22.6 kW and 57.4 kW)