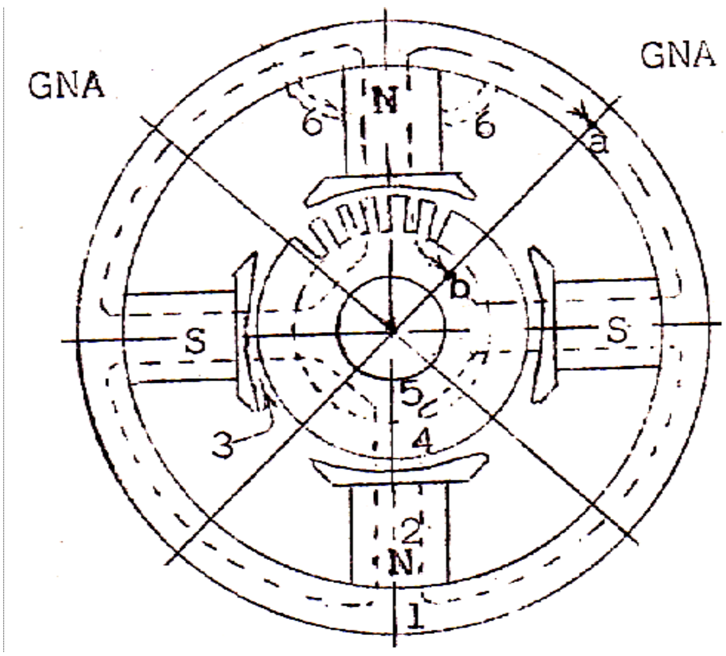
Chapter.4 MAGNETIC CIRCUIT OF A D.C. MACHINE

The different parts of the dc machine magnetic circuit / pole are yoke, pole, air gap, armature teeth and armature core. Therefore, the ampere-turns /pole to establish the required flux in the magnetic circuit is the sum of the ampere-turns required for different parts mentioned above. That is,

AT / pole = $AT_y + AT_p + AT_g + AT_t + AT_c$



1. Yoke, 2. Pole, 3. Air gap, 4. Armature teeth, 5. Armature core, 6. Leakage flux
ab: Mean length of the flux path corresponding to one pole

Magnetic circuit of a 4 pole DC machine

Note:

1. Leakage factor or Leakage coefficient LC.

All the flux produced by the pole ϕ_p will not pass through the desired path i.e., air gap. Some of the flux produced by the pole will be leaking away from the air gap. The flux that passes through the air gap and cut by the armature conductors is the useful flux and that flux that leaks away from the desired path is the leakage flux ϕ_l.

Thus $\phi_P = \phi + \phi_l$

As leakage flux is generally around (15 to 25) % of ϕ ,

$\phi_p = \phi + (0.15 \text{ to } 0.25)\ \phi$

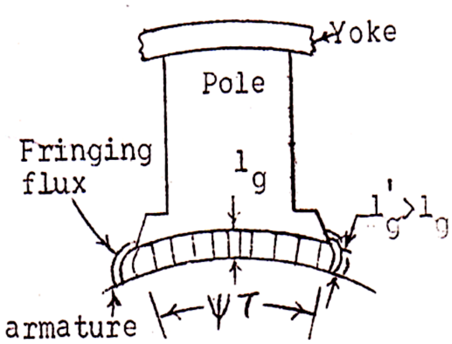
$= LC\ x$

where LC is the Leakage factor or Leakage coefficient and lies between (1.15 to 1.25).

2. Magnitude of flux in different parts of the magnetic circuit
   a) Flux in the yoke $\phi_y = (\phi \times LC)/2$
   b) Flux in the pole $\phi_p = \phi \times LC$
   c) Flux in the air gap = ϕ
   d) Flux in the armature teeth = ϕ
   e) Flux in the armature core = / 2

3. Reluctance of the air gap

Reluctance of the air gap $S = \dfrac{l}{a\,\mu_0\,\mu_r} = \dfrac{l_g}{(\tau\,\psi\,L)\,\mu_0}$ as $\mu_r = 1.0$ for air gap



Flux distribution under a pole

where

l_g = Length of air gap

$\psi\tau$ = Width (pole arc) over which the flux is passing in the air gap

L = Axial length of the armature core

$\psi\tau L$ = Air gap area / pole over which the flux is passing in the air gap

Because of the chamfering of the pole, the length of air gap under the pole varies from l_g at the center of the pole to $l'_g > l_g$ at the pole tip. The length of air gap to be considered for the calculation of air gap reluctance is neither l_g nor l'_g, but has to be a value in between l_g and l'_g. The length of air gap at the tips is generally 1.5 to 2 times the air gap length under the center of the pole.

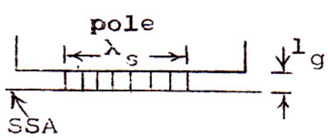
Because of the fringing of flux, the width over which the flux passes through the air gap is not $\psi\tau$ but it is more than that.

The effect of variation in air gap length and fringing of flux can be ignored as the former appears in the numerator and the latter in the denominator of the expression for the reluctance.
While calculating the reluctance of the air gap, effect of the presence of slots and ducts on the armature must also be considered.

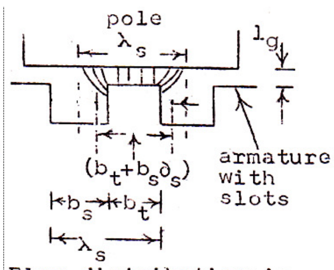
Effect of slots on the reluctance of the air gap

Consider a smooth surface armature (SSA) i.e. having no slots and ducts. Over a slot pitch λ_s, reluctance of the air gap in the presence of smooth surface armature

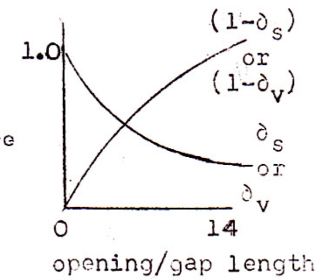
$$S_{SSA} = \frac{l_g}{\lambda_s \, L \, \mu_0} \quad \text{-------} \quad (1)$$



Flux distribution in air gap in the presence of a smooth surface armature



Flux distribution in air gap in the presence of a slotted armature



Carter's fringing coefficient curves

Over the same slot pitch consider a slot and tooth. Because of the crowding effect, the flux instead passing only over the tooth width b_t, passes over some portion of the slot also. Thus the width over which the flux is passing is equal to $(b_t + b_s \delta_s)$ where δ_s is called the Carter's fringing coefficient for slots. It is less than 1.0 and depends on the ratio of slot opening to air gap length and can be obtained from the Carter's fringing coefficient curve.

The reluctance of the air gap in the presence of armature with slots

$$S_{AWS} = \frac{l_g}{(b_t + b_s \, \delta_s) \, L \, \mu_0} \quad \text{-------} \quad (2)$$

Dividing 2 by 1,

$$\frac{S_{AWS}}{S_{SSA}} = \frac{l_g \, / \, (b_t + b_s \, \delta_s) \, L \, \mu_0}{l_g \, / \lambda_s \, L \, \mu_0}$$

$$S_{AWS} = \frac{\lambda_s \times S_{SSA}}{(b_t + b_s \delta_s)}$$

$$= \frac{\lambda_s \times S_{SSA}}{b_t + b_s \delta_s + b_s - b_s}$$ after adding and subtracting b_s in the denominator

$$S_{AWS} = \frac{\lambda_s \times S_{SSA}}{\lambda_s - b_s (1 - \delta_s)} = K_{gs} \times S_{SSA}$$

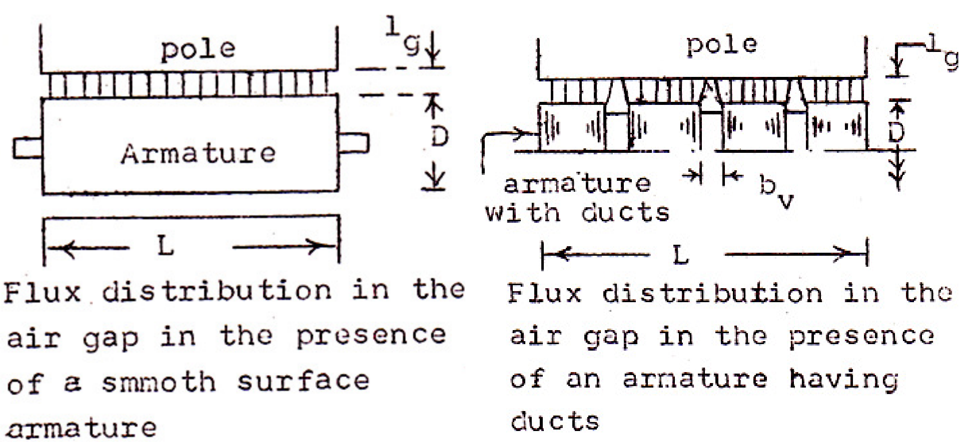
where K_{gs} is called the Carter's gap expansion coefficient for slots and is greater than 1.0.

It is clear from the above expression that the effect of the slots is to increase the reluctance of the air gap by a factor K_{gs} as compared to the reluctance of the air gap in the presence of a smooth surface armature.

Effect of ventilating ducts on the reluctance of the air gap

Consider a smooth surface armature (SSA) i.e. armature having no slots and ducts. Reluctance of the air gap, in the presence of a smooth surface armature

$$S_{SSA} = \frac{l_g}{\pi D L \mu_0} \quad \text{---------} (3)$$



Reluctance of the air gap in the presence of the armature with ducts (AWD)

$$S_{AWD} = \frac{l_g}{\pi D [L - n_v b_v (1 - \delta_v)] \mu_0} \quad \text{---------} (4)$$

where δ_v is the carter's fringing coefficient for ducts. It is less than 1.0 and depends on the ratio opening of the duct to air gap length and is obtained from the Carter's fringing coefficient curve.

Dividing 4 by 3,

$$\frac{S_{AWD}}{S_{SSA}} = \frac{l_g \,/\, \pi D [L - n_v \; b_v \, (1 - \partial_v)] \, \mu_0}{l_g \,/\, \pi D L \mu_0}$$

$$S_{AWD} = \frac{L \times S_{SSA}}{L - n_v b_v (1 - \delta_v)} = K_{gv} \times S_{SSA}$$

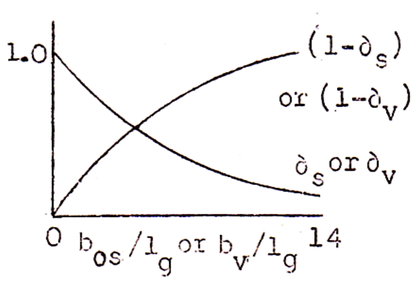
whereK_{gv} is called the Carter's gap expansion coefficient for ducts and is greater than 1.0. Thus the effect of ducts is to increase the reluctance of the air gap by a factor K_{gv} as compared to the reluctance of the air gap in the presence of a smooth surface armature.

Combined effect of slots and ducts on the reluctance of the air gap

The presence of slots and ducts increases the reluctance of the air gap by factors K_{gs} and K_{gv} respectively. Together they increase the reluctance by a factor K_g called the Carter's gap expansion coefficient (or extension coefficient or contraction coefficient). Thus

$$K_g = K_{gs} \times K_{gv} = \frac{\lambda_s}{\lambda_s - b_{os} (1 - \delta_s)} \times \frac{L}{L - n_v \; b_v (1 - \delta_v)}$$

where
b_{os} = opening of the slot
= width of the slot b_s for open type of slot
<b_s for semi-closed slots
= zero for closed slots
δ_s or $(1 - \delta_s)$ = Carter's fringing coefficient for slots and depends on the ratio b_{os} / l_g and can be obtained from the carter's fringing coefficient curve.
δ_v or $(1 - \delta_v)$ = Carter's fringing coefficient for ducts and depends on the ratio b_v / l_g and can be obtained from the carter's fringing coefficient curve.



Carter's fringing coefficient curves

Calculation of ampere-turns per pole for the magnetic circuit of a DC machine

The total ampere turns / pole required for the magnetic circuit of a DC machine to establish flux ϕ,

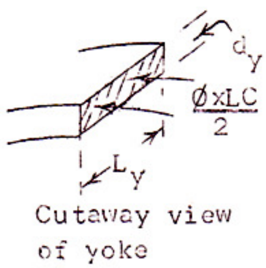
AT / pole = Sum of the ampere turns required to over come the reluctance of the yoke, pole, air gap, armature teeth and armature core

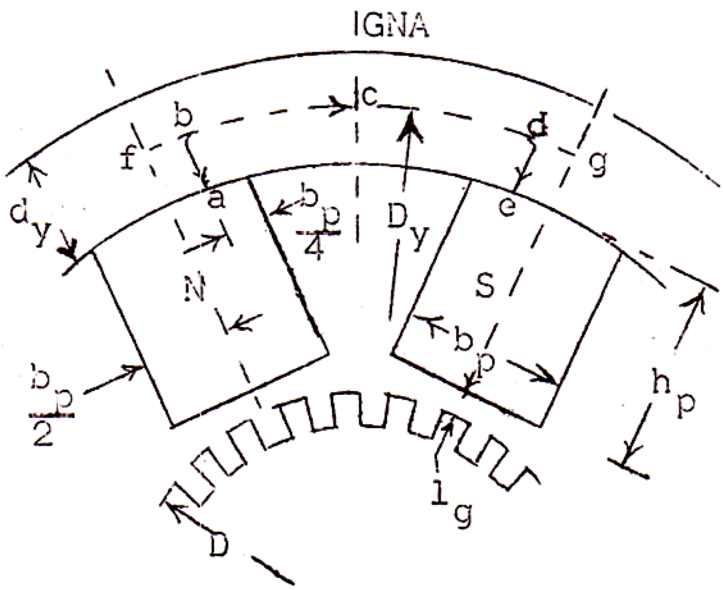
$= AT_y + AT_p + AT_g + AT_t + AT_c$

a) ampere turns for the yoke / pole AT_y :

Flux density in the yoke $B_y = \frac{\phi \times LC / 2}{A_y}$ tesla

Let at_y be the ampere turns per metre, obtained from the magnetization curve corresponding to the yoke material, at B_y.





NOTE:

L_y = Axial length of the yoke

d_y = Depth of the yoke

A_y = Cross-sectional area of yoke = $d_y L_y$

b_p = Width of the pole

D_y = Mean diameter of the yoke = $(D + 2l_g + 2h_p + d_y)$

D = Diameter of the armature

l_g = Length of air gap

h_p = Height of the pole

fg = Pole pitch at mean diameter of the yoke = $\pi D_y / P$

Mean length of the flux path in the yoke

l_y = abc = abcde / 2

= (fg – 2fb + 2ab) /2

$$= \left(\frac{\pi D_y}{P} - \frac{2\, b_p}{4} - \frac{2\, d_y}{2} \right) / 2$$

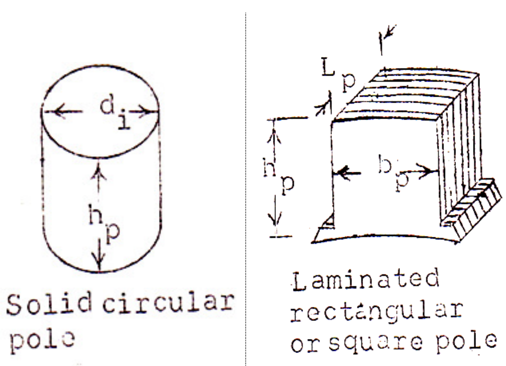
$$= \left(\frac{\pi D_y}{P} - \frac{b_p}{2} - d_y \right) / 2$$

Total ampere-turns for the yoke / pole $AT_y = at_y \times l_y$

b) ampere turns for the pole AT_p :

Flux density in the pole $B_p = \dfrac{\phi \times LC}{A_p}$ tesla

Let at_p be the ampere turns per metre, obtained from the magnetization curve corresponding to the pole material, at B_p.



Note:

L_p = Axial length of the pole

L_{pi} = Net iron length of the pole

h_p = Height of the pole including pole shoe height

$L_{pi} = K_i L_p$

d_i = Diameter of the pole

A_p = Cross-sectional area of the pole

= $b_p L_{pi}$ in case of square or rectangular laminated poles

= $\pi d_i^2 /4$ in case of circular poles

Mean length of the flux path in the pole = pole height h_p

Total ampere turns for the pole / pole $AT_p = at_p \times h_p$

c) ampere turns for the air gap / pole AT_g :

Since flux = mmf or AT / reluctance, ampere turns for the air gap per pole
$AT_g = \phi$ x reluctance.

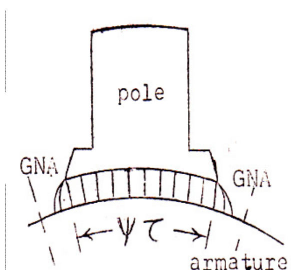
Though the reluctance of the air gap under a pole is $\frac{l_g}{\tau \psi L \mu_0}$, it is to be multiplied by the Carter's gap expansion coefficient $K_g = K_{gs} \times K_{gv}$ in order to account the effect of slots and ducts. Therefore,

$$AT_g = \phi \times \frac{l_g K_g}{\psi \tau L \mu_0} = \frac{l_g K_g B_g}{4\pi x 10^{-7}} = 800{,}000 l_g K_g B_g \text{ (approximately)}$$

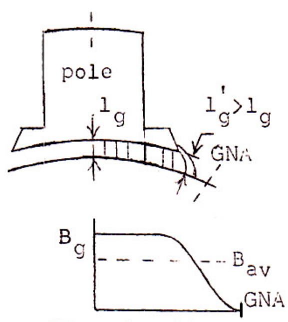
where B_g is the maximum value of the flux density in the air gap along the center line of the pole.
That is,

$$B_g = \frac{}{\psi \tau L} = \frac{}{\frac{\pi DL}{P}\psi} = \frac{P}{\pi DL \psi} = \frac{B_{av}}{\psi}$$

$$= \frac{\text{average value of the flux density } B_{av}}{\text{field form factor } K_f \text{ and is approximately equal to pole enclosure } \psi}$$



Flux distribution in the air gap under a pole



Flux density distribution under a pole

d) ampere turns for the armature teeth / pole AT_t:

Flux density in the armature tooth (in case of a parallel sided slot and tapered tooth) at 1/3 height from the root of the tooth

$$B_{t1/3} = \frac{\phi}{b_{t\,1/3}\, L_i \times S/P}$$

where $b_{t\,1/3}$ = width of the tooth at 1/3 height from the root of the tooth

$$= \frac{\pi\,(D - 4/3\, h_t)}{S} - b_s$$

L_i = Net iron length of the armature core = $K_i\,(L - n_v b_v)$

Let at_t be the ampere turns per metre, obtained from the magnetization curve corresponding to the armature core material, at $B_{t\,1/3}$.

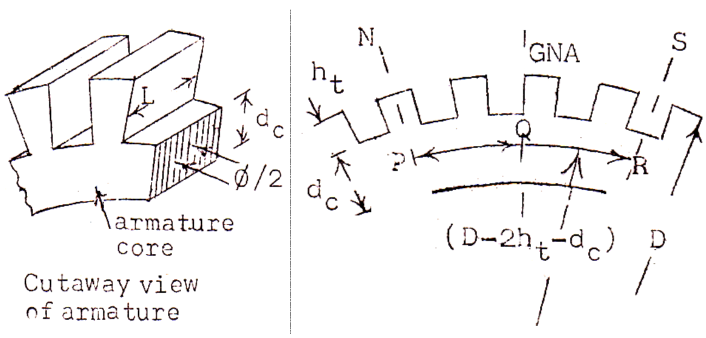
Mean length of the flux path in the tooth = height of the tooth h_t

Total ampere turns for the armature teeth / pole $AT_t = at_t \times h_t$

e) ampere turns for the armature core / pole AT_c :

Flux density in the armature core $B_c = \frac{\phi / 2}{A_c}$ tesla

Let at_c be the ampere turns per metre, obtained from the magnetization curve corresponding to the armature core material, at B_c.



Note:
d_c = Depth of the armature core
A_c = Cross-sectional area of the armature core = $d_c\, L_i$
Mean length of the flux path in the armature core

$$l_c = \frac{PQR}{2} = \frac{\pi\,(D - 2h_t - d_c)}{2P}$$

Total ampere turns for the armature core / pole $AT_c = at_c \times l_c$

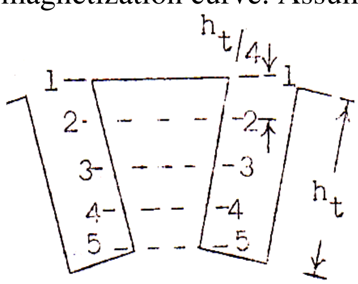
Thus the total ampere-turns required for the magnetic circuit of the DC machine
$AT / pole = AT_y + AT_p + AT_g + AT_t + AT_c$

Methods of calculating the ampere turns for the armature teeth:

For a parallel sided slot, the tooth is tapered and therefore the flux density at each and every section of the tooth will be different. The flux density is least at the air gap surface of the tooth where the flux enters the tooth and maximum at the root of the tooth where the tooth section is minimum. Since the variation of flux density in the tooth is non-linear because of saturation of iron, the calculation of ampere turns becomes difficult.
Different methods available for the calculation of AT_t are

1. Graphical method
2. Simpson's method and
3. $B_{t\,1/3}$ method

Graphical method
In this method the tooth is divided into a number of equal parts and flux density at each tooth section is calculated. Corresponding to each flux density, At / m is obtained from the magnetization curve. Assuming linearity between the sections considered, AT_t is calculated.



Note: h_t = height of the tooth or depth of the slot
b_{t1}, b_{t2}, b_{t3} etc., are the tooth width at different sections 1, 2, 3 etc.

Flux density at section l, $B_{t1} = \dfrac{\phi}{b_{t1} L_i \times S / P}$

Let the ampere turns / metre, obtained from the magnetization curve is H_1 or at_1 at B_{t1}.
Flux density at section 2, $B_{t2} = \dfrac{\phi}{b_{t2} L_i \times S / P}$

Let the ampere turns / metre, obtained from the magnetization curve is H_2 or at_2 at B_{t2}.

Flux density at section 3, $B_{t3} = \dfrac{\phi}{b_{t3} L_i \times S / P}$

Let the ampere turns / metre, obtained from the magnetization curve is H_3 or at_3 at B_{t3}.

Similarly let H_4 be the ampere turns / metre at B_{t4} etc.

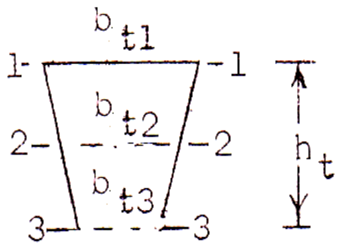
Total ampere turns for the teeth / pole

$$AT_t = \frac{H_1 + H_2}{2} \times \frac{h_t}{n} + \frac{H_2 + H_3}{2} \times \frac{h_t}{n} + \frac{H_3 + H_4}{2} \times \frac{h_t}{n} \text{ etc.,}$$

where n is the number of parts by which the tooth is divided.

Simpson's method

In this method the tooth is divided into two equal parts. The flux density at each section is calculated and the corresponding ampere turns / metre are obtained from the magnetization curve.



Note:b_{t1}, b_{t2}, b_{t3} are the width of the tooth at section 1, 2 and 3

Let H_1 be the AT/m corresponding to the flux density $B_{t1} = \dfrac{\phi}{b_{t1} L_i \times S/P}$ at section 1.

Let H_2 be the AT/m corresponding to the flux density $B_{t2} = \dfrac{\phi}{b_{t2} L_i \times S/P}$ at section 2.

Let H_3 be the AT/m corresponding to the flux density $B_{t3} = \dfrac{\phi}{b_{t3} L_i \times S/P}$ at section 3.

According to Simpson's rule, average ampere turns / m

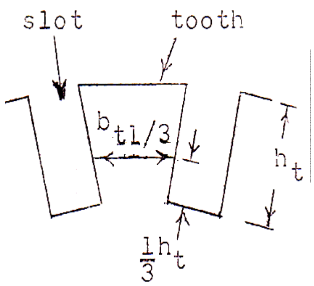
$$H_{av} = \frac{1}{6}(H_1 + 4H_2 + H_3)$$

Total ampere turns for the armature teeth / pole $AT_t = H_{av} \times h_t$

$B_{t\,1/3}$ method

In this method, AT_t is obtained considering the flux density at 1/3 height from the root of the tooth.

Flux density in the tooth at 1/3 height from the root of the tooth $B_{t\,1/3} = \dfrac{\phi}{b_{t\,1/3} L_i \times S/P}$



Let at_t be the ampere turns per metre, obtained from the magnetization curve corresponding to the armature core material, at $B_{t\ 1/3}$.

Total ampere turns for the teeth / pole $AT_t = at_t \times h_t$.

[Note**:** In all the above three methods, the effect of saturation of iron is neglected. In other words all the flux under a slot pitch is assumed to be passing through the tooth only].

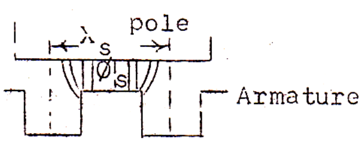
Real and Apparent Flux densities

When the iron is not saturated, reluctance of the iron will be less and all the flux ϕ_s over a slot pitch will be passing through the tooth only. However, when the iron gets saturated, reluctance of the iron increases considerably and the flux over the slot pitch divides itself to take both slot and tooth paths.

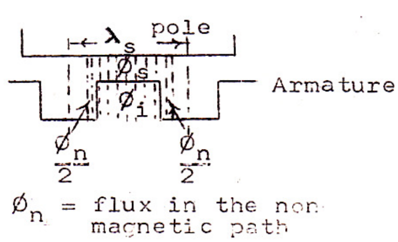
Thus the flux density = $\dfrac{\phi_s}{\text{iron area of the tooth } A_i}$ is not the real or actual flux density

in the tooth, but it is an apparent flux density. The real flux density B_{real} will, however,

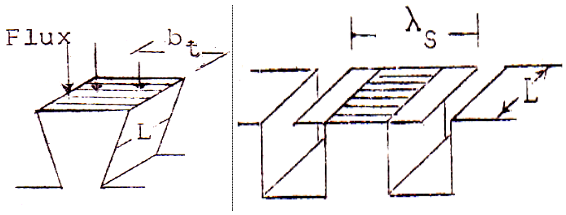
be equal to $\dfrac{\text{flux in the tooth or iron path } \phi_i}{\text{net iron area of the tooth } A_i}$ and will be less than the apparent flux density B_{app}.



Flux distribution when the iron is not saturated



Flux distribution when the iron is saturated



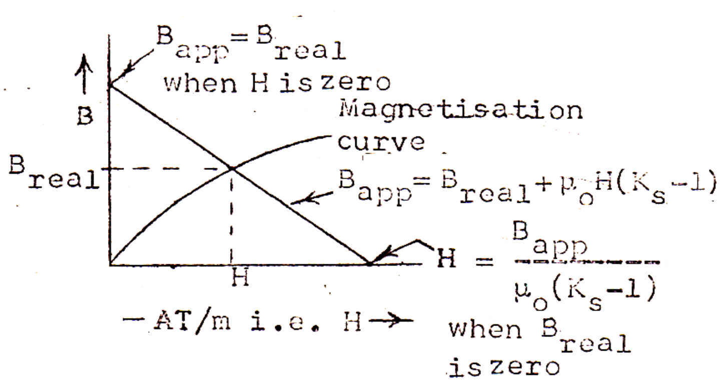
Area of iron or tooth area over which flux is passing $A_i = b_t L_i$
Total area over the slot pitch λ_s L = area of iron A_i + area of non-magnetic path A_n

$$B_{app} = \frac{_s}{A_i} = \frac{_i + _n}{A_i} = \frac{_i}{A_i} + \frac{_n}{A_i} = B_{real} + \frac{_n}{A_n} \times \frac{A_n}{A_i} = B_{real} + B_n K$$

where $B_n = \mu_0\ \mu_r\ H = \mu_0\ H$ is the flux density in the non-magnetic path and K is a constant equal to A_n / A_i . The magnetizing force H is the ampere turns / metre to establish B_{real} or B_n.

Therefore $B_{app} = B_{real} + \mu_0 H K$
If the slot factor $K_s = 1 + K = (1 + \frac{A_n}{A_i}) = \frac{A_i + A_n}{A_i} = \frac{_s L}{b_t L_i}$ then $K = (K_s - 1)$.
Thus $B_{app} = B_{real} + \mu_0 H (K_s - 1)$ and is an equation of straight line.

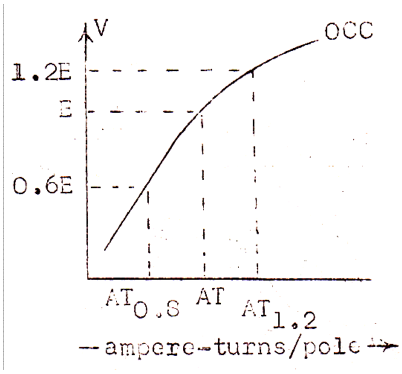
[Note: Since the actual value of flux passing through the slot or tooth is not known, B_n and B_{real} and therefore the AT / m i.e. H to establish B_n or B_{real} are also not known. Hence the equation $B_{app} = B_{real} + \mu_0 H (K_s - 1)$ has two unknowns B_{real} and H. Thus the equation cannot be solved. However the values of B_{real} and H can be found by plotting the above equation on the magnetization curve. The intersection point of the magnetization curve and the straight line provides the values of B_{real} and H.]



The co-ordinates, of the intersection point of magnetization curve and straight line provides the values of B_{real} and H.
Therefore the total ampere-turns for the armature teeth / pole $AT_t = H \times h_t$.

No-load, Magnetization or Open circuit characteristic (OCC)

Since the OCC is a plot of emf induced and AT, ampere-turns for different assumed voltages or flux are found out by calculating AT_y ,AT_p , AT_g, AT_t and AT_c. Accuracy of the curve increases as the number of voltages considered increases.



Information that can be obtained from the open circuit characteristic is,

1) The value of critical field resistance.
2) shunt and series field ampere-turns
3) Effect of armature reaction in conjunction with the internal characteristic.

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