

CHAPTER 2

SYMMETRICAL THREE PHASE FAULTS

[**CONTENTS:** Preamble, transients on a transmission line, short circuit of an unloaded synchronous machine- short circuit currents and reactances, short circuit of a loaded machine, selection of circuit breaker ratings, examples]

2.1 Preamble

in practice, any disturbance in the normal working conditions is termed as a FAULT. The effect of fault is to load the device electrically by many times greater than its normal rating and thus damage the equipment involved. Hence all the equipment in the fault line should be protected from being overloaded. In general, overloading involves the increase of current up to 10-15 times the rated value. In a few cases, like the opening or closing of a circuit breaker, the transient voltages also may overload the equipment and damage them.

In order to protect the equipment during faults, fast acting circuit breakers are put in the lines. To design the rating of these circuit breakers or an auxiliary device, the fault current has to be predicted. By considering the equivalent per unit reactance diagrams, the various faults can be analyzed to determine the fault parameters. This helps in the protection and maintenance of the equipment.

Faults can be symmetrical or unsymmetrical faults. In symmetrical faults, the fault quantity rises to several times the rated value equally in all the three phases. For example, a 3-phase fault - a dead short circuit of all the three lines not involving the ground. On the other hand, the unsymmetrical faults may have the connected fault quantities in a random way. However, such unsymmetrical faults can be analyzed by using the *Symmetrical Components*. Further, the neutrals of the machines and equipment may or may not be grounded or the fault may occur through fault impedance. The three-phase fault involving ground is the most severe fault among the various faults encountered in electric power systems.

2.2 Transients on a transmission line

Now, let us Consider a transmission line of resistance R and inductance L supplied by an ac source of voltage v , such that $v = V_m \sin(\omega t + \alpha)$ as shown in figure 1. Consider the short circuit transient on this transmission line. In order to analyze this symmetrical 3-phase fault, the following assumptions are made:

- The supply is a constant voltage source,
- The short circuit occurs when the line is unloaded and

- The line capacitance is negligible.

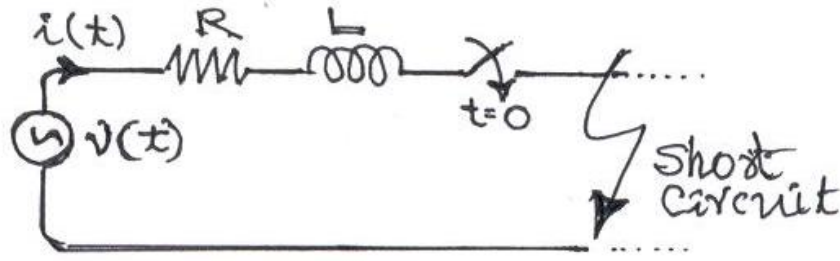


Figure 1. Short Circuit Transients on an Unloaded Line.

Thus the line can be modeled by a lumped R-L series circuit. Let the short circuit take place at $t=0$. The parameter, α controls the instant of short circuit on the voltage wave. From basic circuit theory, it is observed that the current after short circuit is composed of the two parts as under: $i = i_s + i_t$, Where, i_s is the steady state current and i_t is the transient current. These component currents are determined as follows.

Consider, $v = V_m \sin(\omega t + \alpha)$
 $= iR + L (di/dt)$ (2.1)

and $i = I_m \sin(\omega t + \alpha - \theta)$ (2.2)

Where $V_m = \sqrt{2}V$; $I_m = \sqrt{2}I$; $Z_{mag} = \sqrt{[R^2 + (\omega L)^2]}$; $\theta = \tan^{-1}(\omega L/R)$ (2.3)

Thus $i_s = [V_m/Z] \sin(\omega t + \alpha - \theta)$ (2.4)

Consider the performance equation of the circuit of figure 1 under circuit as:

$iR + L (di/dt) = 0$
 i.e., $(R/L + d/dt)i = 0$ (2.5)

In order to solve the equation (5), consider the complementary function part of the solution as: $CF = C_1 e^{(-t/\tau)}$ (2.6)

Where $\tau (= L/R)$ is the time constant and C_1 is a constant given by the value of steady state current at $t = 0$. Thus we have,

$C_1 = -is(0)$
 $= - [V_m/Z] \sin(\alpha - \theta)$
 $= [V_m/Z] \sin(\theta - \alpha)$ (2.7)

Similarly the expression for the transient part is given by:

$i_t = -is(0) e^{(-t/\tau)}$
 $= [V_m/Z] \sin(\theta - \alpha) e^{(-R/L)t}$ (2.8)

Thus the total current under short circuit is given by the solution of equation (1) as [combining equations (4) and (8)],

$$i = i_s + i_t$$

$$= [\sqrt{2V/Z}] \sin(\omega t + \alpha - \theta) + [\sqrt{2V/Z}] \sin(\theta - \alpha) e^{(-R/L)t} \quad (2.9)$$

Thus, i_s is the sinusoidal steady state current called as the *symmetrical short circuit current* and i_t is the unidirectional value called as the *DC off-set current*. This causes the total current to be unsymmetrical till the transient decays, as clearly shown in figure 2.

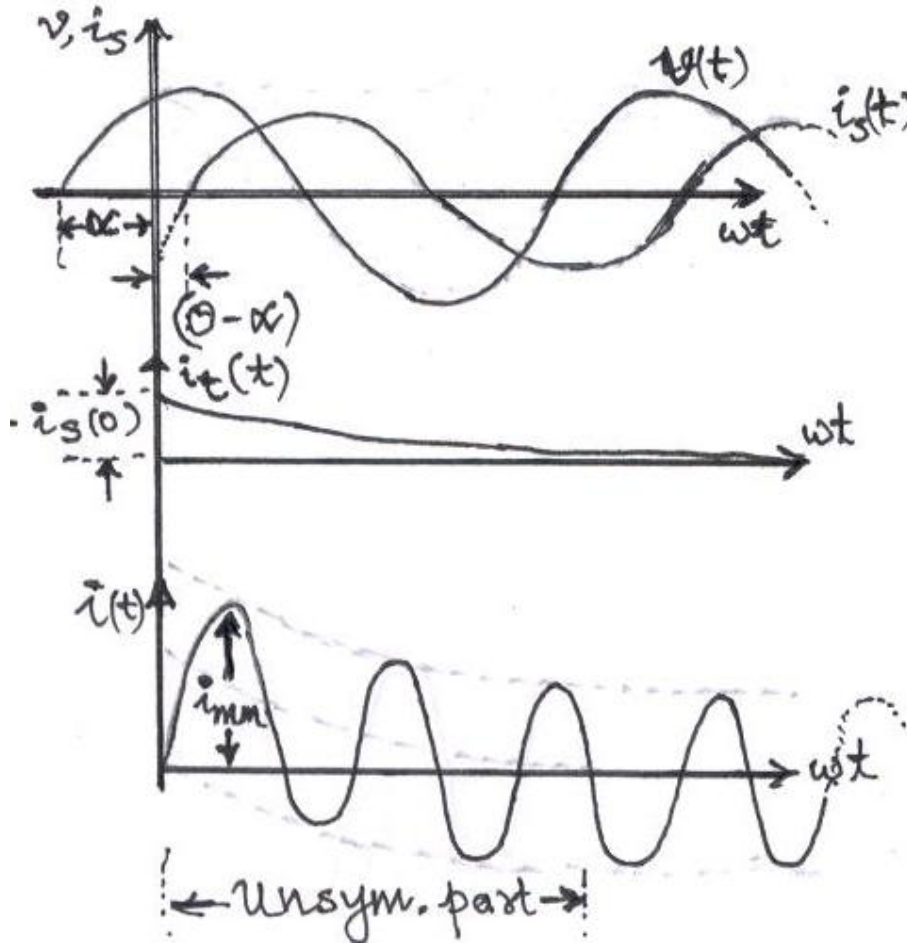


Figure 2. Plot of Symmetrical short circuit current, $i(t)$.

The maximum momentary current, i_{mm} thus corresponds to the first peak. Hence, if the decay in the transient current during this short interval of time is neglected, then we have (sum of the two peak values);

$$i_{mm} = [\sqrt{2V/Z}] \sin(\theta - \alpha) + [\sqrt{2V/Z}] \quad (2.10)$$

now, since the resistance of the transmission line is very small, the impedance angle θ , can be taken to be approximately equal to 90° . Hence, we have

$$i_{mm} = [\sqrt{2V/Z}] \cos \alpha + [\sqrt{2V/Z}] \quad (2.11)$$

This value is maximum when the value of α is equal to zero. This value corresponds to the short circuiting instant of the voltage wave when it is passing through zero. Thus the final expression for the maximum momentary current is obtained as:

$$i_{mm} = 2 [\sqrt{2}V/Z] \quad (2.12)$$

Thus it is observed that the maximum momentary current is twice the maximum value of symmetrical short circuit current. This is referred as the *doubling effect* of the short circuit current during the symmetrical fault on a transmission line.

2.3 Short circuit of an unloaded synchronous machine

2.3.1 Short Circuit Reactances

Under steady state short circuit conditions, the armature reaction in synchronous generator produces a demagnetizing effect. This effect can be modeled as a reactance, X_a in series with the induced emf and the leakage reactance, X_l of the machine as shown in figure 3. Thus the equivalent reactance is given by:

$$X_d = X_a + X_l \quad (2.13)$$

Where X_d is called as the direct axis *synchronous reactance* of the synchronous machine. Consider now a sudden three-phase short circuit of the synchronous generator on no-load. The machine experiences a transient in all the 3 phases, finally ending up in steady state conditions.

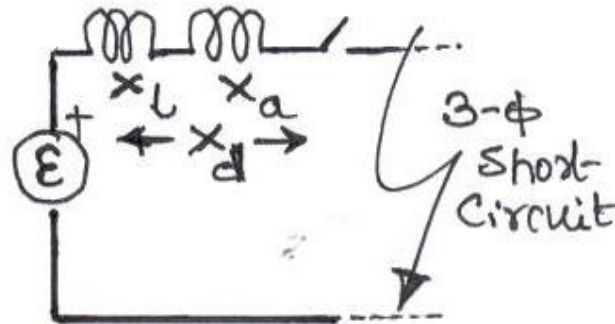


Figure 3. Steady State Short Circuit Model

Immediately after the short circuit, the symmetrical short circuit current is limited only by the leakage reactance of the machine. However, to encounter the demagnetization of the armature short circuit current, current appears in field and damper windings, assisting the rotor field winding to sustain the air-gap flux. Thus during the initial part of the short circuit, there is mutual coupling between stator, rotor and damper windings and hence the corresponding equivalent circuit would be as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d'' = X_l + [1/X_a + 1/X_f + 1/X_{dw}]^{-1} \quad (2.14)$$

Where X_d'' is called as the *sub-transient reactance* of the synchronous machine. Here, the equivalent resistance of the damper winding is more than that of the rotor field winding. Hence, the time constant of the damper field winding is smaller. Thus the damper field effects and the eddy currents disappear after a few cycles.

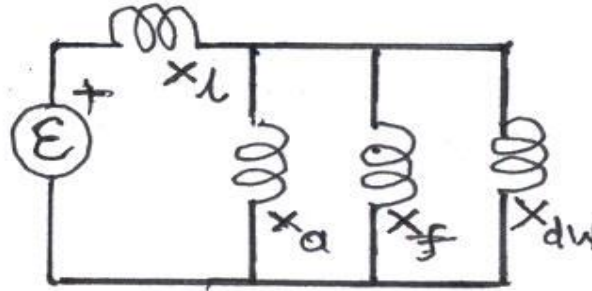


Figure 4. Model during Sub-transient Period of Short Circuit

In other words, X_{dw} gets open circuited from the model of Figure 5 to yield the model as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d' = X_l + [1/X_a + 1/X_f]^{-1} \quad (2.15)$$

Where X_d' is called as the *transient reactance* of the synchronous machine. Subsequently, X_f also gets open circuited depending on the field winding time constant and yields back the steady state model of figure 3.

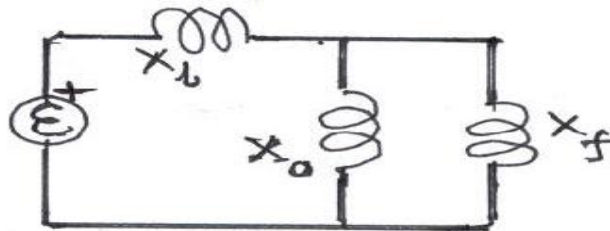


Figure 5. Model during transient Period of Short Circuit

Thus the machine offers a time varying reactance during short circuit and this value of reactance varies from initial stage to final one such that: $X_d > X_d' > X_d''$

2.3.2 Short Circuit Current Oscillogram

Consider the oscillogram of short circuit current of a synchronous machine upon the occurrence of a fault as shown in figure 6. The symmetrical short circuit current can be divided into three zones: the initial sub transient period, the middle transient period and finally the steady state period. The corresponding reactances, X_d'' , X_d' and X_d respectively, are offered by the synchronous machine during these time periods.

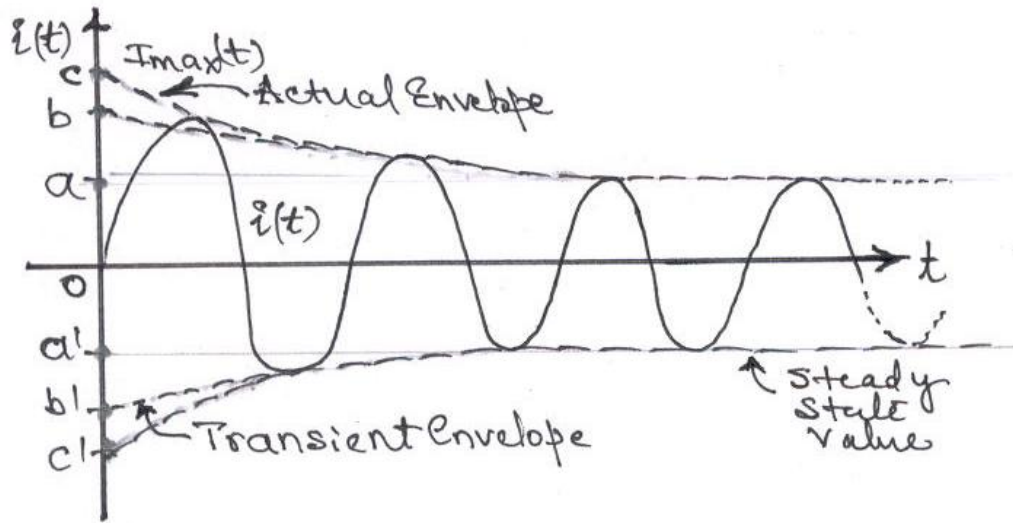


Figure 6. SC current Oscillogram of Armature Current.

The currents and reactances during the three zones of period are related as under in terms of the intercepts on the oscillogram (oa, ob and oc are the y-intercepts as indicated in figure 6):

$$\begin{aligned}
 \text{RMS value of the steady state current} &= I = [oa/\sqrt{2}] = [E_g/X_d] \\
 \text{RMS value of the transient current} &= I' = [ob/\sqrt{2}] = [E_g/X_d'] \\
 \text{RMS value of the sub transient current} &= I'' = [oc/\sqrt{2}] = [E_g/X_d''] \quad (2.16)
 \end{aligned}$$

2.4 short circuit of a loaded machine

In the analysis of section 2.3 above, it has been assumed that the machine operates at no load prior to the occurrence of the fault. On similar lines, the analysis of the fault occurring on a loaded machine can also be considered.

Figure 7 gives the circuit model of a synchronous generator operating under steady state conditions supplying a load current I_L to the bus at a terminal voltage V_t . E_g is the induced emf under the loaded conditions and X_d is the direct axis synchronous reactance of the generator.

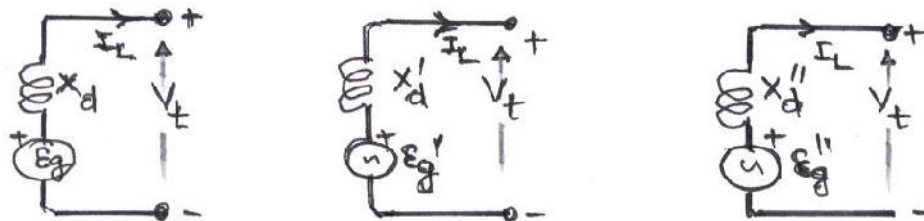


Figure 7. Circuit models for a fault on a loaded machine.

Also shown in figure 7, are the circuit models to be used for short circuit current calculations when a fault occurs at the terminals of the generator, for sub-transient current and transient current values. The induced emf values used in these models are given by the expressions as under:

$$\begin{aligned}
 E_g &= V_t + j I_L X_d = \text{Voltage behind syn. reactance} \\
 E_g' &= V_t + j I_L X_d' = \text{Voltage behind transient reactance} \\
 E_g'' &= V_t + j I_L X_d'' = \text{Voltage behind subtr. Reactance} \quad (2.17)
 \end{aligned}$$

The synchronous motors will also have the terminal emf values and reactances. However, then the current direction is reversed. During short circuit studies, they can be replaced by circuit models similar to those shown in figure 7 above, except that the voltages are given by the relations as under:

$$\begin{aligned}
 E_m &= V_t - j I_L X_d = \text{Voltage behind syn. reactance} \\
 E_m' &= V_t - j I_L X_d' = \text{Voltage behind transient reactance} \\
 E_m'' &= V_t - j I_L X_d'' = \text{Voltage behind subtr. Reactance} \quad (2.18)
 \end{aligned}$$

The circuit models shown above for the synchronous machines are also very useful while dealing with the short circuit of an interconnected system.

2.5 Selection of circuit breaker ratings

For *selection of circuit breakers*, the maximum momentary current is considered corresponding to its maximum possible value. Later, the current to be interrupted is usually taken as symmetrical short circuit current multiplied by an empirical factor in order to account for the DC off-set current. A value of 1.6 is usually selected as the multiplying factor.

Normally, both the generator and motor reactances are used to determine the momentary current flowing on occurrence of a short circuit. The interrupting capacity of a circuit breaker is decided by X_d'' for the generators and X_d' for the motors.

2.6 Examples

Problem #1: A transmission line of inductance 0.1 H and resistance 5 Ω is suddenly short circuited at $t = 0$, at the far end of a transmission line and is supplied by an ac source of voltage $v = 100 \sin (100\pi t + 15^\circ)$. Write the expression for the short circuit current, $i(t)$. Find the approximate value of the first current maximum for the given values of α and θ . What is this value for $\alpha=0$, and $\theta=90^\circ$? What should be the instant of short circuit so that the DC offset current is (i)zero and (ii)maximum?

Solution:

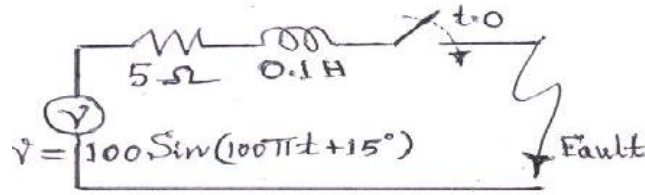


Figure P1.

Consider the expression for voltage applied to the transmission system given by

$$v = V_m \sin(\omega t + \alpha) = 100 \sin(100\pi t + 15^\circ)$$

Thus we get: $V_m = 100$ volts; $f = 50$ Hz and $\alpha = 15^\circ$.

Consider the impedance of the circuit given by:

$$Z = R + j\omega L = 5 + j(100\pi)(0.1) = 5 + j31.416 \text{ ohms.}$$

Thus we have: $Z_{\text{mag}} = 31.8113$ Ohms; $\theta = 80.957^\circ$ and $\tau = L/R = 0.1/5 = 0.02$ seconds.

The short circuit current is given by:

$$\begin{aligned} i(t) &= [V_m/Z] \sin(\omega t + \alpha - \theta) + [V_m/Z] \sin(\theta - \alpha) e^{-(R/L)t} \\ &= [100/31.8113] [\sin(100\pi t + 15^\circ - 80.957^\circ) + \sin(80.957^\circ - 15^\circ) e^{-(t/0.02)}] \\ &= 3.1435 \sin(314.16 t - 65.96) + 2.871 e^{-50t} \end{aligned}$$

Thus we have:

$$\text{i) } i_{\text{mm}} = 3.1435 + 2.871 e^{-50t}$$

where t is the time instant of maximum of symmetrical short circuit current. This instant occurs at $(314.16 t - 65.96) = 90^\circ$; Solving we get, $t = 0.00867$ seconds so that $i_{\text{mm}} = 5$ Amps.

$$\text{ii) } i_{\text{mm}} = 2V_m/Z = \mathbf{6.287 \text{ A}}; \text{ for } \alpha = 0, \text{ and } \theta = 90^\circ \text{ (Also, } i_{\text{mm}} = 2(3.1435) = 6.287 \text{ A)}$$

$$\text{iii) DC offset current} = [V_m/Z] \sin(\theta - \alpha) e^{-(R/L)t}$$

$$= \text{zero, if } (\theta - \alpha) = \text{zero, i.e., } \theta = \alpha, \quad \text{or} \quad \alpha = \mathbf{80.957^\circ}$$

$$= \text{maximum if } (\theta - \alpha) = 90^\circ, \text{ i.e., } \alpha = \theta - 90^\circ, \quad \text{or} \quad \alpha = \mathbf{-9.043^\circ}.$$

Problem #2: A 25 MVA, 11 KV, 20% generator is connected through a step-up transformer- T_1 (25 MVA, 11/66 KV, 10%), transmission line (15% reactance on a base of 25 MVA, 66 KV) and step-down transformer- T_2 (25 MVA, 66/6.6 KV, 10%) to a bus that supplies 3 identical motors in parallel (all motors rated: 5 MVA, 6.6 KV, 25%). A circuit breaker-A is used near the primary of the transformer T_1 and breaker-B is used near the motor M_3 . Find the symmetrical currents to be interrupted by circuit breakers A and B for a fault at a point P, near the circuit breaker B.

Solution:

Consider the SLD with the data given in the problem statement. The base values are selected as under:

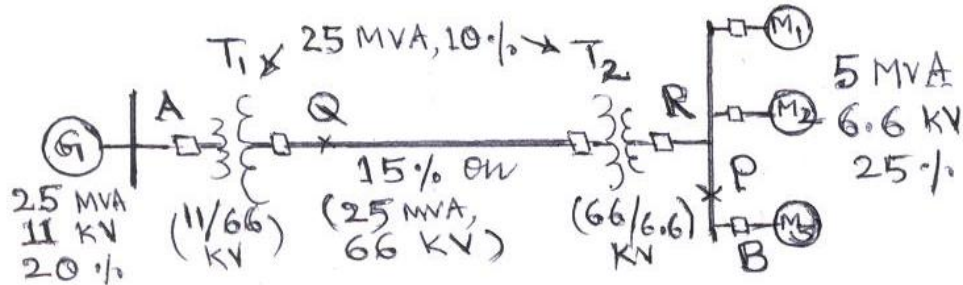


Figure P2(a)

Selection of bases:

$S_b = 25 \text{ MVA}$ (common); $V_b = 11 \text{ KV}$ (Gen. circuit)- chosen so that then $V_b = 66 \text{ KV}$ (line circuit) and $V_b = 6.6 \text{ KV}$ (Motor circuit).

Pu values:

$X_g = j0.2 \text{ pu}$, $X_{t1} = X_{t2} = j0.1 \text{ pu}$; $X_{m1} = X_{m2} = X_{m3} = j0.25(25/5) = j1.25 \text{ pu}$; $X_{line} = j0.15 \text{ pu}$.

Since the system is operating at no load, all the voltages before fault are 1 pu. Considering the pu reactance diagram with the faults at P, we have:

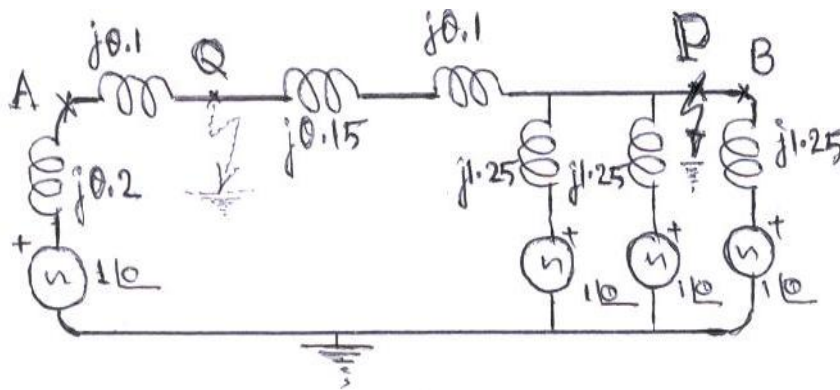


Figure P2(b)

Current to be interrupted by circuit breaker A = $1.0 / j[0.2+0.1+0.15+0.1]$

$$= -j 1.818 \text{ pu} = -j 1.818 (25/[\sqrt{3}(11)]) = -j 1.818 (1.312) \text{ KA} = \mathbf{2.386 \text{ KA}}$$

And Current to be interrupted by breaker B = $1/j1.25 = -j 0.8 \text{ pu}$

$$= -j0.8 (25/[\sqrt{3}(6.6)]) = -j0.8 (2.187) \text{ KA} = \mathbf{1.75 \text{ KA.}}$$

Problem #3: Two synchronous motors are connected to a large system bus through a short line. The ratings of the various components are: Motors(each)= 1 MVA, 440 volts, 0.1 pu reactance; line of 0.05 ohm reactance and the short circuit MVA at the bus of the large system is 8 at 440 volts. Calculate the symmetrical short circuit current fed into a three-phase fault at the motor bus when the motors are operating at 400 volts.

Solution:

Consider the SLD with the data given in the problem statement. The base values are selected as under:

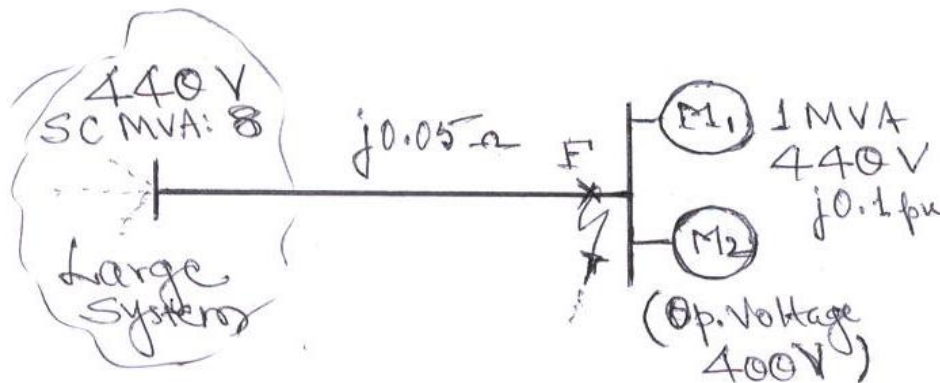


Figure P3.

$S_b = 1 \text{ MVA}$; $V_b = 0.44 \text{ KV}$ (common)- chosen so that $X_m(\text{each})=j0.1 \text{ pu}$, $E_m = 1.0\angle 0^\circ$, $X_{\text{line}}=j0.05 (1/0.44^2) = j 0.258 \text{ pu}$ and $X_{\text{large-system}} = (1/8) = j 0.125 \text{ pu}$.

Thus the prefault voltage at the motor bus; $V_t = 0.4/0.44 = 0.909\angle 0^\circ$,

Short circuit current fed to the fault at motor bus ($I_f = YV$);

$$I_f = [0.125 + 0.258]^{-1} + 2.0 \} 0.909 = [20.55 \text{ pu}] [1000/(\sqrt{3}(0.4))] \\ = 20.55 (1.312) \text{ KA} = \mathbf{26.966 \text{ KA}}$$

Problem #4: A generator-transformer unit is connected to a line through a circuit breaker. The unit ratings are: Gen.: 10 MVA, 6.6 KV, $X_d'' = 0.1 \text{ pu}$, $X_d' = 0.2 \text{ pu}$ and $X_d = 0.8 \text{ pu}$; and Transformer: 10 MVA, 6.9/33 KV, $X_1 = 0.08 \text{ pu}$; The system is operating on no-load at a line voltage of 30 KV, when a three-phase fault occurs on the line just beyond the circuit breaker. Determine the following:

- (i) Initial symmetrical RMS current in the breaker,
- (ii) Maximum possible DC off-set current in the breaker,
- (iii) Momentary current rating of the breaker,
- (iv) Current to be interrupted by the breaker and the interrupting KVA and
- (v) Sustained short circuit current in the breaker.

Solution:

Consider the base values selected as **10 MVA**, **6.6 KV** (in the generator circuit) and $6.6(33/6.9) = \mathbf{31.56}$ KV(in the transformer circuit). Thus the base current is:

$$I_b = 10 / [\sqrt{3}(31.56)] = \mathbf{0.183} \text{ KA}$$

The pu values are: $X_d'' = 0.1$ pu, $X_d' = 0.2$ pu and $X_d = 0.8$ pu; and $X_{Tr} = 0.08 (6.9/6.6)^2 = 0.0874$ pu; $V_t = (30/31.6) = 0.95 \angle 0^0$ pu.

Initial symmetrical RMS current = $0.95 \angle 0^0 / [0.1 + 0.0874] = 5.069$ pu = **0.9277** KA;

Maximum possible DC off-set current = $2 (0.9277) = \mathbf{1.312}$ KA;

Momentary current rating = $1.6(0.9277) = \mathbf{1.4843}$ KA; (assuming 60% allowance)

Current to be interrupted by the breaker (5 Cycles) = $1.1(0.9277) = \mathbf{1.0205}$ KA;

Interrupting MVA = $3(30) (1.0205) = \mathbf{53.03}$ MVA;

Sustained short circuit current in the breaker = $0.95 \angle 0^0 (0.183) / [0.8 + 0.0874] = \mathbf{0.1959}$ KA.

2.7 Exercises for Practice

PROBLEMS

1. The one line diagram for a radial system network consists of two generators, rated 10 MVA, 15% and 10 MVA, 12.5 % respectively and connected in parallel to a bus bar A at 11 KV. Supply from bus A is fed to bus B (at 33 KV) through a transformer T_1 (rated: 10 MVA, 10%) and OH line (30 KM long). A transformer T_2 (rated: 5 MVA, 8%) is used in between bus B (at 33 KV) and bus C (at 6.6 KV). The length of cable running from the bus C up to the point of fault, F is 3 KM. Determine the current and line voltage at 11 kV bus A under fault conditions, when a fault occurs at the point F, given that $Z_{cable} = 0.135 + j 0.08$ ohm/kM and $Z_{OH-line} = 0.27 + j 0.36$ ohm/kM. [Answer: 9.62 kV at the 11 kV bus]

2. A generator (rated: 25MVA, 12. KV, 10%) supplies power to a motor (rated: 20 MVA, 3.8 KV, 10%) through a step-up transformer (rated:25 MVA, 11/33 KV, 8%), transmission line (of reactance 20 ohms) and a step-down transformer (rated:20 MVA, 33/3.3 KV, 10%). Write the pu reactance diagram. The system is loaded such that the motor is drawing 15 MW at 0.9 leading power factor, the motor terminal voltage being 3.1 KV. Find the sub-transient current in the generator and motor for a fault at the generator bus. [Answer: $I_g'' = 9.337$ KA; $I_m'' = 6.9$ KA]

3. A synchronous generator feeds bus 1 and a power network feed bus 2 of a system. Buses 1 and 2 are connected through a transformer and a line. Per unit reactances of the components are: Generator(bus-1):0.25; Transformer:0.12 and Line:0.28. The power network is represented by a generator with an unknown reactance in series. With the generator on no-load and with 1.0 pu voltage at each bus, a three phase fault occurring on bus-1 causes a current of 5 pu to flow into the fault. Determine the equivalent reactance of the power network. [Answer: $X = 0.6$ pu]

4. A synchronous generator, rated 500 KVA, 440 Volts, 0.1 pu sub-transient reactance is supplying a passive load of 400 KW, at 0.8 power factor (lag). Calculate the initial symmetrical RMS current for a three-phase fault at the generator terminals.

[Answer: $S_b=0.5$ MVA; $V_b=0.44$ KV; load= $0.8 \angle -36.9^0$; $I_b=0.656$ KA; $I_f=6.97$ KA]

OBJECTIVE TYPE QUESTIONS

1. When a 1-phase supply is across a 1-phase winding, the nature of the magnetic field produced is
 - a) Constant in magnitude and direction
 - b) Constant in magnitude and rotating at synchronous speed
 - c) Pulsating in nature
 - d) Rotating in nature
2. The damper windings are used in alternators to
 - a) Reduce eddy current loss
 - b) Reduce hunting
 - c) Make rotor dynamically balanced
 - d) Reduce armature reaction
3. The neutral path impedance Z_n is used in the equivalent sequence network models as
 - a) Z_n^2
 - b) Z_n
 - c) $3 Z_n$
 - d) An ineffective value
4. An infinite bus-bar should maintain
 - a) Constant frequency and Constant voltage
 - b) Infinite frequency and Infinite voltage
 - c) Constant frequency and Variable voltage
 - d) Variable frequency and Variable voltage
5. Voltages under extra high voltage are
 - a) 1KV & above
 - b) 11KV & above
 - c) 132 KV & above
 - d) 330 KV & above

[Ans.: 1(c), 2(b), 3(c), 4(a), 5(d)]