UNIT 3  Friction and Belt Drives

Structure

Definitions

Types of Friction
Laws of friction
Friction in Pivot and Collar Bearings

Belt Drives

Flat Belt Drives
Ratio of Belt Tensions
Centrifugal Tension
Power Transmitted

INTRODUCTION

When a body moves or tends to move on another body, a force appears between the surfaces. This force is called force of friction and it acts opposite to the direction of motion. The force arises from the fact that the surfaces, though planed and made smooth, have ridges and depressions that interlock and the relative movement is resisted (Fig. 3.1). Thus, the force of friction on a body is parallel to the sliding surfaces and acts in a direction opposite to that of the sliding body (Fig. 3.2). The magnitude of this force depends on the roughness of surfaces.
There are phenomena, where it is necessary to reduce the force of friction whereas in some cases it must be increased.

**Reduction of force of friction**

In case of lathe slides, journal bearings, etc., where the power transmitted is reduced due to friction, it has to be decreased by the use of lubricated surfaces.

**Increase of force of friction**

In processes where the power itself is transmitted through friction, attempts are made to increase it to transmit more power. Examples are friction clutches and belt drives etc. Even the tightness of a nut and bolt is dependent mainly on the force of friction.

**KINDS OF FRICTION**

Usually, three kinds of friction, depending upon the conditions of surfaces are considered.

**Dry Friction**

Dry friction is said to occur when there is relative motion between two completely unlubricated surfaces. It is further divided into two types:
(a) Solid Friction: When the two surfaces have a sliding motion relative to each other, it is called a solid friction.

(b) Rolling Friction: Friction due to rolling of one surface over another (e.g. ball and roller bearings) is called rolling friction.

Skin or Greasy Friction

When the two surfaces in contact have a minute thin layer of lubricant between them, it is known as skin or greasy friction. Higher spots on the surface break through the lubricant and come in contact with the other surface. Skin friction is also termed as boundary friction (Fig. 3.3).

![Fig. 3.3 Dry, greasy and film friction](image)

Film Friction

When the two surfaces in contact are completely separated by a lubricant, friction will occur due to the resistance of motion between the lubricant and the surfaces in contact with it. This is known as film friction or viscous friction (Fig. 3.3).

LAWS OF FRICTION

Experiments have shown that the force of **solid friction** (Static friction)

- is directly proportional to normal reaction between the two surfaces
• opposes the motion between the surfaces
• depends upon the materials of the two surfaces
• is independent of the area of contact
• is independent of the velocity of sliding

**Kinetic friction**

• The force of friction always acts in a direction opposite to that in which the body is moving.
• The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces.
• The force of friction is independent of the relative velocity between the two surfaces in contact, but it decreases slightly with increase in velocity.
• The force of friction increases with reversal of motion.
• The COF changes slightly when the temperature changes.

**Fluid friction**

• The force of friction is almost independent of the load.
• The force of friction reduces with the increase of temperature of the lubricant.
• The force of friction depends upon the type and viscosity of the lubricant.
• The force of friction is independent of the nature of surfaces.
• The frictional force increases with the increase in the relative velocity of the frictional surfaces.

![Fig. 3.3](www.getmyuni.com)
From Fig. 3.3, Block of weight $W$ placed on horizontal surface. Forces acting on block are its weight and reaction of surface $N$.

![Fig. 3.3](image)

Fig. 3.3

From Fig. 3.4, Small horizontal force $P$ applied to block. For block to remain stationary, in equilibrium, a horizontal component $F$ of the surface reaction is required. $F$ is a static-friction force.

As $P$ increases, the static-friction force $F$ increases as well until it reaches a maximum value $F_m$.

$$F_m = \mu_S N$$

![Fig. 3.4](image)

Fig. 3.4

Further increase in $P$ causes the block to begin to move as $F$ drops to a smaller kinetic-friction force $F_k$.

$$F_k = \mu_K N$$

![Fig. 3.5](image)

Fig. 3.5
COEFFICIENT OF FRICTION

Let a body of weight \( W \) rest on a smooth and dry plane surface. Under the circumstances, the plane surface also exerts a reaction force \( R_n \) on the body which is normal to the plane surface. If the plane surface considered is horizontal, \( R_n \) would be equal and opposite to \( W \) (Fig. 3.6(a)).

![Fig 3.6](image)

Let a small horizontal force \( F \) be applied to the body to move it on the surface (Fig. 3.6(b)). So long the body is unable to move, the equilibrium of the body provides,

\[
R_n = W \quad \text{and} \quad F = F'
\]

Where \( F' \) is the horizontal force resisting the motion of the body. As the force \( F \) is increased, the resistive force \( F' \) also increases accordingly. \( F' \) and \( R_n \), the friction and the normal reaction forces can also be combined into a single reaction force \( R \) inclined at an angle \( \theta \) to the normal. Thus

\[
R \cos \theta = W \quad \text{and} \quad R \sin \theta = F
\]
At a moment, when the force $F$ would just move the body, the value of $F'$ or $R \sin \theta$ (equal to $F$) is called the static force of friction. Angle $\theta$ attains the value $\phi$ and the body is in equilibrium under the action of three forces (Fig. 3.6(c))

$F$, in the horizontal direction

$W$, in the vertical downward direction, and

$R$, at an angle $\phi$ with the normal (inclined towards the force of friction).

According to the first law of friction,

$$F' \propto R_n$$

$$F' = \mu R_n$$

Where $\mu$ is known as the coefficient of friction.

Or

$$\mu = \frac{F'}{Rn}$$

Also in fig 8.1 c, $\tan \phi = \frac{F'}{Rn}$

Or

$$\tan \phi = \frac{\mu R_n}{Rn} = \mu$$

The angle $\phi$ is known as the limiting angle of friction, or simply the angle of friction.

Now, if the body moves over the plane surface, it is observed that the friction force will be slightly less than the static friction force. As long as the body moves with a uniform velocity, the force $F$ required for the motion of the body will be equal to the force of friction on the body. However, if the velocity is to increase, additional force will be needed to accelerate the body. "Thus, while the body is in motion, it can be written that

$$\tan \phi = \mu$$
Where $\phi$ is approximately the limiting angle of friction.

Also, no movement is possible until the angle of reaction $R$ with the normal becomes equal to the limiting angle of friction or until $\tan \phi = \mu$.

**Four situations can occur when a rigid body is in contact with a horizontal surface:**

**Fig. 3.7 (a) No friction, ($P_x = 0$)**

$P_x < F_m$

$F = 0$

$N = P + W$

**Fig. 3.7 (b) No motion, ($P_x < F_m$)**

$F = P_x$

$F < \mu_s N$

$N = P_y + W$

$$
\tan \phi_s = \frac{F_m}{N} = \frac{\mu_s}{N}
$$

$$
\tan \phi_s = \mu_s
$$
Fig. 3.7 (c)

Motion impending,
\[(P_x = F_m)\]

\[
\tan \phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}
\]

\[
\tan \phi_k = \mu_k
\]

Approximate values of static coefficient of friction for dry (un-lubricated) and greasy lubricated surfaces are given in Table 1:

<table>
<thead>
<tr>
<th>Materials</th>
<th>Dry</th>
<th>Greasy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard Steel On Hard Steel</td>
<td>0.42</td>
<td>0.029 - 0.108</td>
</tr>
<tr>
<td>Mild Steel On Mild Steel</td>
<td>0.57</td>
<td>0.09-0.19</td>
</tr>
<tr>
<td>Mild Steel On Cast Iron</td>
<td>0.24</td>
<td>0.09-0.116</td>
</tr>
<tr>
<td>Wood On Metal</td>
<td>0.2-0.6</td>
<td>-</td>
</tr>
<tr>
<td>Metal On Stone</td>
<td>0.3-0.7</td>
<td>-</td>
</tr>
<tr>
<td>Metal On Leather</td>
<td>0.3-0.6</td>
<td>-</td>
</tr>
<tr>
<td>Wood On Leather</td>
<td>0.2-0.5</td>
<td>-</td>
</tr>
<tr>
<td>Cast Iron On Cast Iron</td>
<td>0.3-0.34</td>
<td>0.065-0.070</td>
</tr>
<tr>
<td>Rubber On Ice</td>
<td>0.05-0.2</td>
<td>-</td>
</tr>
</tbody>
</table>
Pivot and Collar Bearing

The rotating shafts are frequently subjected to axial thrust. These shafts can be kept in correct axial position if bearing surfaces are provided. The bearing surfaces which are flat or conical carry the axial thrust. The bearing surfaces placed at the end of a shaft are known as pivots.

The pivot may have a flat surface or a conical surface or truncated conical surface as shown in

Fig 3.8 (a), (b) and (c) respectively.

![Fig. 3.8](image)

The bearing surfaces provided at any position along the shaft (but nut at the end of the shaft), to carry the axial thrust, is known as collar. The surface of the collar may be flat (normal to the axis of shaft) or of conical shape as shown in Fig. 3.9 (a) and (b) respectively. The collar bearings are also known as thrust bearing.
For a new bearing, the contact between the shaft and bearing may be good over the whole surface. This means that the pressure over the rubbing surfaces may be assumed as uniformly distributed. But when the bearing becomes old, all parts of the rubbing surfaces will not move with the same velocity and hence the wear will be different at different radii. The pressure distribution will not be uniform. The rate of wear of surfaces depends upon the pressure and the rubbing velocities between the surfaces.

The design of bearings is based on the following assumptions though neither of them is strictly true:

(1) the pressure is uniformly distributed over the bearing surfaces, and

(2) the wear is uniform over the bearing surface.

The power lost, due to friction in pivot and collar bearings, are calculated on the above two assumptions.

**Flat Pivot**

The bearing surface placed at the end of the shaft is known as pivot. If the surface is flat as shown in Fig. 3.10, then the bearing surface is called flat pivot or
foot-step. There will be friction along the surface of contact between the shaft and bearing. The power lost can be obtained by calculating the torque.

Let

\[ W = \text{Axial load, or load transmitted to the bearing surface}, \]
\[ R = \text{Radius of pivot}, \]
\[ \mu = \text{Co-efficient of friction}, \]
\[ p = \text{Intensity of pressure in N/m}^2, \]
\[ T = \text{Total frictional torque}. \]

Consider a circular ring of radius \( r \) and thickness \( dr \) as shown in Fig. 3.10.

Area of ring = \( 2\pi r dr \)

We will consider the two cases, namely

(1) case of uniform pressure over bearing surface and
(2) case of uniform wear over bearing surface.

\textbf{(1) case of Uniform Pressure}

When the pressure is assumed to be uniform over the bearing surface, then intensity of pressure \( (p) \) is given by

\[
P = \frac{\text{Axial load}}{\text{area of cross section}} = \frac{W}{\pi R^2}
\]

\text{…… (1)}

Now let us find the load transmitted to the ring and also frictional torque on the ring.

Load transmitted to the ring

= Pressure on the ring \( x \) Area of ring
Frictional force* on the ring
=μX load on ring
=μxp 2Π rdr
Frictional torque on the ring = Friction force x Radius of ring.

Frictional torque,\(dT= \mu xp 2\Pi rdr \times r\)
=2\(\Pi \mu pr^2 \, dr\)
The total frictional torque (T) will be obtained by integrating the above equation from 0 to R.

\[
\therefore \text{Total frictional torque, } \quad T = \int_0^R 2\pi \mu pr^2 \, dr
\]

\[
= 2\pi \mu p \int_0^R r^2 \, dr \quad (\because \mu \text{ and } p \text{ are constant})
\]

\[
= 2\pi \mu p \left[ \frac{r^3}{3} \right]_0^R = 2\pi \mu p \left[ \frac{R^3}{3} \right] = \frac{2}{3} \pi \mu p R^3
\]

\[
= \frac{2}{3} \pi \times \mu \times \frac{W}{\pi R^2} \times R^3 \quad (\because \text{from (1), } p = \frac{W}{\pi R^2})
\]

\[
= \frac{2}{3} \mu W R
\]

\[
\therefore \text{power lost in friction } = T \times \omega
\]

\[
= T \times \frac{2\pi N}{60} \quad (\because \omega = \frac{2\pi N}{60})
\]

\[
= \frac{2\pi NT}{60}
\]

(2) Case of Uniform Wear
For the uniform wear of the bearing surface, the load transmitted to the various, circular rings should be same (or should be constant). But load transmitted to any circular ring is equal to the product of pressure and area of the ring. hence for uniform wear, the product of pressure and area of ring should be constant.
Area of the ring is directly proportional to the radius of the ring. Hence for uniform wear, the product of pressure and radius should be constant or \( p \times r \) constant.

Hence for uniform wear, we have

\[
P \times r = \text{constant} \quad (\text{say C})
\]

\[
\therefore \quad p = \frac{C}{r} \quad \ldots (1)
\]

Now we know that load transmitted to the ring

\[
= \text{pressure} \times \text{area of ring}
\]

\[
= p \times 2\pi rdr
\]

\[
= \frac{C}{r} \times 2\pi rdr \quad (\because \text{from (1), } p = \frac{C}{r})
\]

\[
= 2\pi C \, dr \quad \ldots (2)
\]

Total load transmitted to the bearing, is obtained by integrating the above equation from 0 to \( R \).

\[
\therefore \quad \text{Total load transmitted to the bearing,}
\]

\[
= \int_{0}^{R} 2\pi C \, dr = 2\pi C \int_{0}^{R} \, dr = 2\pi C [r]_{0}^{R}
\]

\[
= 2\pi CR
\]

But total load transmitted to the bearing, = \( W \)

\[
\therefore \quad 2\pi CR = W
\]

\[
C = \frac{W}{2\pi R} \quad \ldots (3)
\]
Now frictional force on the ring

\[ = \mu \times \text{load on ring} \]

\[ = \mu \times 2\pi C \, dr \quad \text{(∵ from (2), load on ring} = 2\pi C \, dr) \]

Hence frictional torque on the ring,

\[ dT = \text{frictional force on ring} \times \text{radius} \]

\[ = \mu \times 2\pi C \, dr \times r \]

\[ \therefore \text{total frictional torque}, \quad T = \int_0^R dT \]

\[ = \int_0^R \mu \times 2\pi C \, r \, dr \]

\[ = \mu \times 2\pi C \int_0^R r \, dr \quad (\mu \text{ and } C \text{ are constant}) \]

\[ = \mu \times 2\pi C \left[ \frac{r^2}{2} \right]_0^R = \mu \times 2\pi C \times \frac{R^2}{2} \]

\[ = \mu \times 2\pi \times \frac{W}{2\pi R} \times \frac{R^2}{2} \quad (\because \text{from (3), } C = \frac{W}{2\pi R}) \]

\[ T = \frac{1}{2} \times \mu \times W \times R \]

\[ \therefore \text{power lost in friction} = T \times \omega = \frac{2\pi NT}{60} \]

**Problem1.** Find the power lost in friction assuming (1) uniform pressure and (ii) uniform wear when a vertical shaft of 100 mm diameter rotating at 150 r.p.m. rests on a flat end foot step bearing. The coefficient of friction is equal to 0.05 and shaft carries a vertical load of 15 kN.

Sol. Given Diameter, \( D = 100 \text{mm} = 0.1 \text{m} \quad R = 0.1/2 = 0.05 \text{m} \)
Speed, \( N = 150 \) r.p.m, Friction co-efficient, \( \mu = 0.05 \) Load, 
\[ W = 15 \text{ kN} = 15 \times 10^3 \text{ N} \]

(1) Power lost in friction assuming uniform pressure

For uniform pressure, frictional torque is given by equation as

\[
T = \frac{2}{3} \mu W R
\]

\[
= \frac{2}{3} \times 0.05 \times 15 \times 10^3 \times 0.05 \text{ Nm} = 25 \text{ Nm}
\]

\[ \therefore \text{Power lost in friction} = \frac{2\pi NT}{60} \]

\[
= \frac{2 \times 150 \times 25}{60} W = 392.7 \text{ W} \quad \text{Ans.}
\]

(2) Power lost in friction assuming uniform wear,

For uniform wear, the frictional torque is given by equation

\[
T = \frac{1}{2} \mu W R
\]

\[
= \frac{1}{2} \times 0.05 \times 15 \times 10^3 \times 0.05 \text{ Nm} = 18.75 \text{ Nm}
\]

\[ \therefore \text{Power lost in friction} = \frac{2\pi NT}{60} \]

\[
= \frac{2 \times 150 \times 18.75}{60} = 294.5 \text{ W} \quad \text{Ans.} \]
Conical Pivot

The bearing surface placed at the end of a shaft and having a conical surface, is known as conical pivot as shown in Fig. 3.11(a).

Let $W =$ Axial load, or load transmitted to the bearing surface

$\mu =$ Coefficient of friction

$R =$ Radius of shaft

$\alpha =$ Semi-angle of the cone

$p =$ Pressure intensity normal to the cone surface.

Consider a circular ring of radius $r$ and thickness $dr$.

The actual thickness of the sloping ring $\frac{dr}{\sin \alpha}$ as shown in Fig. 3.11 (b) in which $AB = dr$ on enlarged scale, angle $ACB = \alpha$ and sloping length of ring $AC = \frac{AB}{\sin \alpha} = \frac{dr}{\sin \alpha}$

Area of ring along conical surface

$= 2\pi r \times \text{actual thickness of sloping ring}$

$= 2\pi r \times \frac{dr}{\sin \alpha}$

Now we will consider two Cases namely

(1) Case of uniform pressure

(2) Case of uniform wear.
(1) Case of Uniform Pressure

Let us first find the load acting on the circular ring, normal to the conical surface.

'. Load on the ring normal to conical surface

= pressure x area of ring along conical surface

= \( p \times 2\pi r \times \frac{dr}{\sin \alpha} \)

Vertical component of the above load

= \( \left[ p \times 2\pi r \times \frac{dr}{\sin \alpha} \right] \times \sin \alpha \)

= \( p \times 2\pi r \times dr \)

\( \therefore \) total vertical load transmitted to the bearing

= \( \int_0^R p \times 2\pi r \times dr \)

= \( p \times 2\pi \int_0^R r \times dr \) \( \because \) pressure is uniform and hence constant

= \( p \times 2\pi \left[ \frac{r^2}{2} \right]_0^R = p \times 2\pi \frac{R^2}{2} = p \times \pi R^2 \)

but total vertical load transmitted is also = \( W \)

\( \therefore \) \( W = p \times \pi R^2 \) \( \ldots (1) \)

Also \( p = \frac{w}{\pi R^2} \) \( \ldots (2) \)

The above equation shows that pressure intensity is independent of the angle of the cone.

Now the frictional force on the ring along the conical surface

= \( \mu \times \text{load on ring normal to conical surface} \)
= \mu \times \left[ p \times 2\pi r \times \frac{dr}{\sin \alpha} \right]

Moment of this frictional force about the shaft axis (dT)

= Frictional torque on the ring

= Frictional force x Radius

= \mu \times \left[ p \times 2\pi r \times \frac{dr}{\sin \alpha} \right] \times r

Total moment of the frictional force about the shaft axis or total frictional torque on the conical surface is obtained by integrating the above equation from 0 to R.

Total frictional torque

\[ T = \int_{0}^{R} \mu \times \left[ p \times 2\pi r \times \frac{dr}{\sin \alpha} \right] \times r \]

= \frac{2\pi \times \mu \times p}{\sin \alpha} \int_{0}^{R} r^2 dr \quad (\because \mu, p \text{ and } \alpha \text{ are constant})

= \frac{2\pi \times \mu \times p}{\sin \alpha} \left[ \frac{r^3}{3} \right]_{0}^{R} = \frac{2\pi \times \mu \times p}{\sin \alpha} \times \frac{R^3}{3}

= \frac{2\pi \times \mu}{\sin \alpha} \times \frac{W}{\pi R^2} \times \frac{R^3}{3} \quad (\because \text{from (2), } p = \frac{W}{\pi R^2})

\[ = \frac{2}{3} \times \frac{\mu W R}{\sin \alpha} \]

\therefore \text{power lost in friction} = \frac{2\pi NT}{60}

(2) case of uniform wear

From equation, for uniform wear we know that

\[ P \times r = \text{constant} \quad (\text{say } C) \]

\therefore \quad P \times r = C
Or \[ p = \frac{c}{r} \]

The equation for total vertical load transmitted to the bearing

\[ = \int_0^R p \times 2\pi r \times dr \]

\[ = \int_0^R \frac{c}{r} \times 2\pi r \times dr \]

\[ = 2 \pi \times C \int_0^R dr = 2 \pi \times C [r]_0^R \]

\[ = 2 \pi \times C \times R \]

But total vertical load transmitted to the bearing is also equal to W

\[ \therefore \quad W = 2 \pi \times C \times R \]

Or \[ C = \frac{W}{2\pi R} \]

Now the frictional torque on the ring is given by the equation

\[ dT = \mu x p \times 2\pi r \times \frac{dr}{\sin \alpha} \times r \]

\[ = \mu x \frac{c}{r} \times 2\pi r \times \frac{dr}{\sin \alpha} \times r \quad (\because p = \frac{c}{r} \text{ for uniform wear}) \]

\[ = 2\pi \mu C \times r \times \frac{dr}{\sin \alpha} \]

\[ = 2\pi \mu \left\{ \frac{W}{2\pi R} \right\} r \times \frac{dr}{\sin \alpha} \quad (\because C = \frac{W}{2\pi R}) \]

\[ \therefore \quad \text{total frictional torque} \]

\[ T = \int_0^R dT = \int_0^R 2\pi \mu \left\{ \frac{W}{2\pi R} \right\} r \times \frac{dr}{\sin \alpha} \]

\[ = 2\pi \mu \left\{ \frac{W}{2\pi R} \right\} \times \frac{1}{\sin \alpha} \times \int_0^R r \, dr \]
\[ = 2\pi \mu \frac{W}{2\pi R} \times \frac{1}{\sin \alpha} \times \frac{R^2}{2} \]

\[ = \frac{1}{2} \times \frac{\mu W R}{\sin \alpha} \]

\[ \therefore \text{ power lost in friction} = \frac{2\pi NT}{60} \]

**Truncated Conical Pivot**

Fig. 3.12 shows the truncated conical pivot of external and internal radii as \( r_1 \) and \( r_2 \).

![Fig 3.12](image)

**Case of Uniform Pressure**

Total vertical load transmitted to the bearing is attained from equation in which the limits of integration arc from \( r_2 \) to \( r_1 \).

Total vertical load transmitted to the bearing

\[ = \int_{r_2}^{r_1} p \times 2\pi r \times dr \]
\[ = p \cdot 2\pi \int_{r_2}^{r_1} r \cdot dr \]

(p is constant for uniform pressure)

\[ = p \cdot 2\pi \left[ \frac{r^2}{2} \right]_{r_2}^{r_1} = p \cdot 2\pi \left[ \frac{r_1^2 - r_2^2}{2} \right] \]

But total vertical load = \( W \)

\[ W = p \cdot 2\pi \left[ \frac{r_1^2 - r_2^2}{2} \right] = p \cdot \pi [r_1^2 - r_2^2] \]

\[ p = \frac{W}{\pi [r_1^2 - r_2^2]} \]

\[ T = \int_{r_2}^{r_1} 2\pi \mu \mu \cdot C \cdot r \cdot \frac{dr}{\sin \alpha} \cdot r \]

\[ = \frac{2\pi \mu p}{\sin \alpha} \int_{r_2}^{r_1} r^2 dr \]

\[ = \frac{2\mu p \cdot r^3}{\sin \alpha} \left[ \frac{r_1}{3} \right]_2 \]

\[ = \frac{2\pi \mu \cdot W}{\sin \alpha \cdot \pi (r_1^2 - r_2^2)} \cdot \left[ \frac{r_1^3 - r_2^3}{3} \right] \]

...... 
......

where \[ p = \frac{W}{\pi (r_1^2 - r_2^2)} \]

\[ T = \frac{2}{3} \cdot \frac{\mu W}{\sin \alpha} \cdot \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right) \]
(2) case of uniform wear

For uniform wear pxr=C

The total vertical load transmitted to the bearing is obtained from equation in which limits of integration are from r2 and r1.

Total vertical load transmitted

\[
= \int_{r_2}^{r_1} p \cdot 2\pi r \cdot dr
\]

\[
= \int_{r_2}^{r_1} \frac{c}{r} \cdot 2\pi r \cdot dr
\]

\[
= 2\pi C \int_{r_2}^{r_1} dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C [r_1 - r_2]
\]

But total vertical load \( = W \)

\[
W = 2\pi C [r_1 - r_2]
\]

\[
c = \frac{W}{2\pi [r_1 - r_2]}
\]

The total frictional torque for uniform wear is obtained by integrating the equation r2 to r1.

\[
\therefore \text{Total frictional torque,}
\]

\[
T = \int_{r_2}^{r_1} 2\pi \mu \cdot C \cdot r \cdot \frac{dr}{\sin a}
\]
Problem 2 A conical pivot with angle of cone as 120°, supports a vertical shaft of diameter 300 mm. It is subjected to a load of 20 KN. The coefficient of friction is 0.05 and the speed of shaft is 210 r.p.m. Calculate the power lost in friction assuming (14) uniform pressure and (2) uniform wear.

Sol. Given:

\[ 2\alpha = 120^\circ \quad \Rightarrow \quad \alpha = 60^\circ ; \]
\[ D = 300 \text{ mm} = 0.3 \text{ m} \quad \Rightarrow \quad R = 0.15 \text{ m} ; \]
\[ W = 20 \text{ KN} = 20 \times 10^3 \text{ N} ; \quad \mu = 0.05 ; \]
\[ N = 210 \text{ r.p.m.} \]

(1) Power lost in friction for uniform pressure

The frictional torque is given by the equation as

\[ T = \frac{2}{3} \times \frac{\mu WR}{\sin \alpha} \]
\[ = \frac{2}{3} \times \frac{0.05 \times 0.15 \times 20 \times 10^3}{\sin 60^\circ} = 115.53 \text{ Nm} \]
\text{Power lost} = \frac{2\pi NT}{60}

= \frac{2\pi \times 210 \times 115.53}{60} = 2540.6 \text{ W} = 2.54 \text{ KW Ans.}

(2) Power lost in friction for uniform wear

The friction torque is given by equation as

\[ T = \frac{1}{2} \mu WR \sin \alpha \]

\[ = \frac{1}{2} \times 0.05 \times 0.15 \times 20 \times 10^3 \times \frac{\sin 60^\circ}{\sin 60^\circ} = 86.60 \text{ Nm} \]

\[ \therefore \text{ power lost} = \frac{2\pi NT}{60} \]

\[ = \frac{2\pi \times 210 \times 86.6}{60} = 1904.4 \text{ W} = 1.9044 \text{ KW. Ans.} \]

\textbf{Problem 3} A load of 25 KN is supported by a conical pivot with angle of cone as 120°. The intensity of pressure is not to exceed 350 KN/m². The external radius is 2 times the internal radius. The shaft is rotating at 180 r.p.m. and coefficient of friction is 0.05. find the power absorbed in friction assuming uniform pressure.

Sol. Given.

Load, \( W = 25 \text{ KN} = 25 \times 10^3 \text{ N}; \) angle of cone, \( 2\alpha = 120^\circ \text{ or } \alpha = 60^\circ \)

Pressure, \( P = 350 \text{ KN/m}^2 = 350 \times 10^3 \text{ N/m}^2 \); external radius = 2 x internal radius

Hence \( r_1 = 2r_2 \); speed, \( N = 180 \text{ r.p.m} \); and \( \mu = 0.05 \)

Using equation for uniform pressure, we get
\[ P = \frac{W}{\pi (r_1^2 - r_2^2)} \]

Or \[ 350 \times 10^3 = \frac{25 \times 10^3}{\pi [(2)r_2^2 - r_2^2]} \]

Or \[ [(2)r_2^2 - r_2^2] = \frac{25}{\pi \times 350} \]

\[ = 0.02273 \]

Or \[ 3r_2^2 = 0.02273 \]

Or \[ r_2 = \sqrt{\frac{0.02273}{3}} = 0.087 \text{ m} \]

\[ \therefore r_1 = 2 \ r_2 = 2 \times 0.087 = 0.174 \text{ m} \]

To find the power absorbed in friction, first calculate the total frictional torque when pressure is uniform.

Frictional torque when pressure is uniform is given by equation as

\[ T = \frac{2}{3} \times \frac{\mu W}{\sin \alpha} \times \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right) \]

\[ = \frac{2}{3} \times \frac{0.05 \times 25 \times 10^3}{\sin 60} \left( \frac{0.174^3 - 0.087^3}{0.174^2 - 0.087^2} \right) \]

\[ = 962.278 \left( \frac{0.005268 - 0.0006585}{0.03027 - 0.007569} \right) \]

\[ = 962.278 \left( \frac{0.0046095}{0.0227} \right) \]

\[ = 195.37 \text{ Nm} \]

\[ \text{power absorbed in friction,} \]

\[ P = \frac{2 \pi N T}{60} = \frac{2 \pi \times 180 \times 195.37}{60} = 3682.6 \ W = 3.6826 \text{ KW. } \text{ Ans} \]
**Flat collar**

The bearing surface provided at any position along the shaft (but not at the end of the shaft), to carry axial thrust is known as collar which may be flat or conical. If the surface is flat, then bearing surface is known as flat collar as shown in fig.3.13 the collar bearings are also known as thrust bearings, the power lost in friction can be obtained by calculating the torque.

Let

\[ r_1 = \text{external radius of collar} \]
\[ r_2 = \text{internal radius of collar} \]
\[ p = \text{intensity of pressure} \]
\[ W = \text{axial load or total load transmitted to the bearing surface} \]
\[ \mu = \text{coefficient of friction} \]
\[ T = \text{total frictional torque} \]
Consider a circular ring of radius \( r \) and thickness \( dr \) as shown in fig 3f.4a

\[
\text{Area of ring} = 2\pi rd\]

\[
\text{Load on the ring} = \text{Pressure} \times \text{Area of ring} = p \times 2\pi r \, dr \quad \ldots \ldots (1)
\]

Frictional force on the ring = \( \mu \times p \times 2\pi r \, dr \)

\[
\text{Frictional torque on the ring} \, dT = \text{Frictional force} \times \text{Radius} = (\mu \times p \times 2\pi r \, dr) \times r = 2\mu p\pi r^2 \, dr
\]

Total frictional torque

\[
T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} 2\mu p\pi r^2 \, dr \quad \ldots \ldots \ldots \ldots \ldots (2)
\]

(1) Uniform pressure

\[
p = \text{constant}
\]

Total load transmitted to the bearing

\[
= \int_{r_2}^{r_1} \text{Load on ring} = \int_{r_2}^{r_1} p \times 2\pi r \, dr = p \times 2\pi \int_{r_2}^{r_1} r \, dr
\]
\[
\begin{align*}
T &= \frac{W}{\pi (r_1^2 - r_2^2)} \\
&= 2\pi \mu \left[ \frac{r_1^3 - r_2^3}{3} \right]
\end{align*}
\]

Total torque is given by equation (2),

\[
T = 2\pi \mu \left( \frac{r_1^3 - r_2^3}{3} \right)
\]

\[
T = \frac{2}{3} \mu W \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)
\]

power lost in friction, \( p = \frac{2\pi NT}{60} \)
(2) uniform wear

\[ p \times r = \text{constant} \]

\[ p = \frac{C}{r} \]

Total load transmitted to the bearing

\[ W = \int_{r_2}^{r_1} p \times 2\pi r dr \]

\[ = \int_{r_2}^{r_1} \frac{C}{r} \times 2\pi r dr \]

\[ = 2\pi C \int_{r_2}^{r_1} dr \]

\[ = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2) \]

\[ = \frac{W}{2\pi (r_1 - r_2)} \ldots \ldots \ldots \ldots (3) \]

Total frictional torque is given by equation (2)

\[ T = \int_{r_2}^{r_1} 2\pi \mu r^2 dr \]

\[ = 2\pi \mu \int_{r_2}^{r_1} pr^2 dr \]

\[ = 2\pi \mu \int_{r_2}^{r_1} \frac{C}{r} r^2 dr \]

\[ = 2\pi \mu \int_{r_2}^{r_1} Crdr = 2\pi \mu C \int_{r_2}^{r_1} r dr \]
Problem 4 In a collar thrust bearing the external and internal radii are 250 mm and 150 mm respectively. The total axial load is 50 KN and shaft is rotating at 150 rpm. The coefficient of friction is equal to 0.05. Find the power lost in friction assuming uniform pressure.

Sol. Given

External radius, \( r_1 = 250 \text{ mm} = 0.25 \text{ m} \)

Internal radius, \( r_2 = 150 \text{ mm} = 0.15 \text{ m} \)

Total axial load, \( W = 50 \text{ KN} = 50 \times 10^3 \text{ N} \)

Speed, \( N = 150 \text{ rpm} \)

Coefficient of friction, \( \mu = 0.05 \)

For uniform pressure, the total frictional torque is given by equation

\[
T = \frac{2}{3} \mu W \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)
\]
If the axial load on bearing is too great then the bearing pressure on the collar will be more then the limiting bearing pressure which is approximately equal to 400KN/m². Hence to reduce the intensity of pressure on collar, two or more collars are used (or multi collars used) as shown in fig 3.14.

\[
\frac{2 \times 0.05 \times 50 \times 10^3 \left( \frac{0.25^3 - 0.15^3}{0.25^2 - 0.15^2} \right)}{3}
\]

\[= 510.42 Nm\]

\[\text{power lost in friction,}\]

\[\text{power lost in friction, } p = \frac{2\pi NT}{60}\]

\[= \frac{2\pi \times 150 \times 510.42}{60} = 8.0176 kW. \text{ ans}\]

**Multi-collars**

If the axial load on bearing is too great then the bearing pressure on the collar will be more then the limiting bearing pressure which is approximately equal to 400KN/m². Hence to reduce the intensity of pressure on collar, two or more collars are used (or multi collars used) as shown in fig 3.14.
(1) \(n=\frac{\text{Total load}}{\text{Load permissible on one collar}}\)

(3) \(p=\frac{\text{load}}{\text{number of collar} \times \text{Area of one collar}} = \frac{W}{n \times \pi (r_1^2 - r_2^2)}\)

(3) Total torque transmitted remains constant i.e.

\[ T = \frac{2}{3} \mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \]

**Problem 5** In a thrust bearing the external and internal radii of the contact surfaces are 210 mm and 160 mm respectively. The total axial load is 60 KN and coefficient of friction = 0.05. The shaft is rotating at 380 rpm. Intensity of pressure is not to exceed 350 KN/ m².

Calculate:

1. Power lost in overcoming the friction and
2. Number of collars required for the thrust bearing.

**Sol.** Given:

External radius, \(r_1 = 210 \text{ mm} = 0.21 \text{ m}\)

Internal radius, \(r_2 = 160 \text{ mm} = 0.16 \text{ m}\)

Total axial load, \(W = 60 \text{ KN} = 60 \times 10^3 \text{ N.}\)

Coefficient of friction, \(\mu = 0.05\)
Speed, \( N = 380 \text{ rpm} \).

Intensity of pressure, \( P = 350 \text{ KN/m}^2 = 350 \times 10^3 \text{ N/m}^2 \)

Here the power lost in overcoming the friction is to be determined. Also no assumption is mentioned.

Hence it is safe to assume uniform pressure.

1. Power lost in overcoming friction

For uniform pressure, total frictional torque is given by equation as

\[
T = \frac{2}{3} \mu W \left( \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)
\]

\[
= \frac{2}{3} \times 0.05 \times 60 \times 10^3 \left( \frac{0.21^3 - 0.16^3}{0.21^2 - 0.16^2} \right)
\]

\[
= 558.378 \text{ Nm}
\]

\[
power \ lost \ in \ friction, \ p = \frac{2\pi NT}{60}
\]

\[
= \frac{2\pi \times 380 \times 558.378}{60} = 22.2198 \text{ Kw. ans}
\]

(2) Numbers of collars required

\[
number \ of \ collars, = \frac{\text{total load}}{\text{load per collar}}
\]

Now load per collar for uniform pressure is obtained from equation
\[ p = \frac{w^*}{\pi(r_1^2 - r_2^2)} \]

where \( w^* \) is the load per collar

\[ W = p \times \pi(r_1^2 - r_2^2) \]
\[ = 350 \times 10^3 \times \pi(0.21^2 - 0.16^2) \]
\[ = 20341.8 N \]

number of collars, \( n \) = \( \frac{\text{total load}}{\text{load per collar}} \)

\[ = \frac{W}{w^*} = \frac{60 \times 10^3}{20341.8} = 2.95 = 3 \text{ collars ans} \]
Belt Drives

Introduction

Usually, power is transmitted from one shaft to another by means of belts, ropes, chains and gears, the salient features of which are as follows:

1. Belts, ropes and chains are used where the distance between the shafts is large. For small distances, gears are preferred.

2. Belts, ropes and chains are flexible type of connectors, i.e., they are bent easily.

3. The flexibility of belts and ropes is due to the property of their materials whereas chains have a number of small rigid elements having relative motion between the two elements.

4 Belts and ropes transmit power due to friction between them and the pulleys. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.

5 Belts and ropes are strained during motion as tensions are developed in them.

6. Owing to slipping and straining action, belts and ropes are not positive type of drives, i.e. their velocity ratios are not constant. On the other hand, chains and gears have constant velocity ratios.

BELT DRIVES

To transmit power from one shaft to another, pulleys are mounted on the two shafts. The pulleys are then connected by an endless belt passing over the pulleys. The connecting belt is kept in tension so that motion of one pulley is transferred to the other without slip. The speed of the driven shaft can be varied by varying the diameters of the two pulleys.
For an un-stretched belt mounted on the pulleys, the outer and the inner faces become in tension and compression respectively (Fig. 3.15). In between there is a neutral section which has no tension or compression. Usually, this is considered at half the thickness of the belt. The effective radius of rotation of a pulley is obtained by adding half the belt thickness to the radius of the pulley.
A belt may be of rectangular section, known as flat belt [Fig. 3.16(a)] or of trapezoidal section, known as V-belt [Fig. 3.16(b)]. In case of a flat belt, the rim of the pulley is slightly crowned which helps to keep the belt running centrally on the pulley rim. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove. Owing to wedging action, V-belts need little adjustment and transmit more power, without slip, as compared to flat belts. Also, a multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity. Generally, these are more suitable for shorter centre distances.

For power transmission by ropes, grooved pulleys are used [Fig. 3.16(c)]. The rope is gripped on its sides as it bends down in the groove reducing the chances of slipping. Pulleys with several grooves can also be employed to increase the capacity of power transmission [Fig. 3.16(d)]. These may be connected in either of the two ways:

1. Using a continuous rope passing from one pulley to the other and back again to the same pulley in the next groove and so on.

2. Using one rope for each pair of grooves.

The advantage of using continuous rope is that the tension in it is uniformly distributed. However, in case of belt failure, the whole drive is put out of action. Using one rope for each groove poses difficulty in tightening the ropes to the same extent but with the advantage that the system can continue its operation even if a rope fails. The repair can be undertaken when it is convenient.

Rope drives are usually, preferred for long centre distances between the shafts, ropes being cheaper as compared to belts. These days, however, long distances are avoided and thus the use of ropes has been limited.
OPEN AND CROSSED BELT DRIVES

1. Open-Belt Drive

An open belt drive is used when the driven pulley is desired to be rotated in the same direction as the driving pulley as shown in Fig.3.15.

Generally, the centre distance for an open-belt drive is 14 to 16 m. If the centre distance is too large, the belt whips, i.e. vibrates in a direction perpendicular to the direction or motion. For very shorter centre distances, the bell slip increases. Both these phenomena limit the use of belts for power transmission.

While transmitting power, one side of the belt is more tightened (known as tight side) than the other (known as slack side). In case of horizontal drives, it is always desired that the tight side is at the lower side of two pulleys. This is because the sag of the belt will be more on the upper side than the lower side. This slightly increases the angles of wrap of the belt on the two pulleys than if the belt had been perfectly straight between the pulleys. In case the tight side is at the upper side, the sag will be greater at the lower side, reducing the angle of wrap and slip could occur earlier. This ultimately affects the power to be transmitted.

2. Crossed-Belt Drive

A crossed-belt drive is adopted when the driven pulley is to be rotated in the opposite direction to that of the driving pulley (Fig. 3.17).
A crossed-belt drive can transmit more power than an open-belt drive as the angle of wrap is more. However, the belt has to bend in two different planes and it wears out more.

**VELOCITY RATIO**

Velocity ratio is the ratio of speed of the driven pulley to that of the driving pulley.

Let \( N_1 \) = rotational speed of the driving pulley

\( N_2 \) = rotational speed of the driven pulley

\( D_1 \) = diameter of the driving pulley

\( D_2 \) = diameter of the driven pulley

\( t \) = thickness of the belt

Neglecting any slip between the belt and the pulleys and also considering the belt to be inelastic,

\[
\text{Speed of belt on driving pulley} = \text{speed of belt on driven pulley} = \left( D_1 + 2 \frac{t}{2} \right) N_1 = \left( D_2 + 2 \frac{t}{2} \right) N_2
\]

Or

\[
VR = \frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}
\]
SLIP

The effect of slip is to decrease the speed of belt on the driving shaft and to decrease the speed of the driven shaft.

Let \( \omega_1 = \) Angular velocity of the driving pulley

\[
\omega_2 = \text{Angular velocity of the driven pulley}
\]

\( S_1 = \) percentage slip between the driving pulley and the belt

\( S_2 = \) percentage slip between the driven pulley and the belt

\( S = \) total percentage slip,

Peripheral speed of driving pulley = \( \omega_1 \left( \frac{D_1+t}{2} \right) \)

Speed of belt on the driving pulley = \[ \omega_1 \left( \frac{D_1+t}{2} \right) \left( \frac{100-S_1}{100} \right) \]

This is also the speed of the belt on the driven pulley.

Peripheral speed of driven pulley = \[ \left[ \omega_1 \left( \frac{D_1+t}{2} \right) \left( \frac{100-S_1}{100} \right) \left( \frac{100-S_2}{100} \right) \right] \]

As \( S \) is the total percentage slip,

Peripheral speed of driven shaft = \[ \omega_1 \left( \frac{D_1+t}{2} \right) \left( \frac{100-S}{100} \right) \]

\[
\left[ \omega_1 \left( \frac{D_1+t}{2} \right) \left( \frac{100-S_1}{100} \right) \left( \frac{100-S_2}{100} \right) \right] = \omega_1 \left( \frac{D_1+t}{2} \right) \left( \frac{100-S}{100} \right)
\]

\[
\frac{(100-S_1)(100-S_2)}{100X100} = \frac{100-S}{100}
\]

Or \( (100 - S_1) (100 - S_2) = 100 (100 - S) \)
Or \[ 10000 = 100 \, S_2 - 100 \, S_1 + S_1 \, S_2 = 10000 - 100S \]

Or \[ 100 \, S = 100 \, S_1 + 100 \, S_2 - S_1 \, S_2 \]

Or \[ S = S_1 + S_2 - 0.01S_1 \, S_2 \]

Effect of slip is to reduce the velocity ratio,

\[ VR = \frac{N_2}{N_1} = \left( \frac{D_1 + t}{D_2 + t} \right) \left( \frac{100 - S}{100} \right) \]

Also it is to be remembered that slip will first occur on the pulley with smaller angle of lap i.e. on the smaller pulley.

**Example**

A shaft runs at 800 rpm and drives another shaft at 150 rpm through belt drive. The diameter of the driving pulley is 600 mm. Determine the diameter of the driven pulley in the following cases:

i. Neglecting belt thickness,

ii. Taking belt thickness as 5 mm,

iii. Assuming for case (ii) a total slip of 4%, and

iv. Assuming for case (ii) a slip of 2% on each pulley.

Solution:

\[ N_1 = 80 \, \text{rpm} \quad \quad \quad D_1 = 600 \, \text{mm} \]

\[ N_2 = 150 \, \text{rpm} \]

i. \[ \frac{N_2}{N_1} = \frac{D_1}{D_2} \quad \text{or} \quad \frac{150}{80} = \frac{600}{D_2} \quad \text{or} \quad D_2 = 320 \, \text{mm} \]

ii. \[ \frac{N_2}{N_1} = \left( \frac{D_1 + t}{D_2 + t} \right) \quad \text{or} \quad \frac{150}{80} = \left( \frac{600 + 5}{D_2 + 5} \right) \quad \text{or} \quad D_2 = 317.7 \, \text{mm} \]

iii. \[ \frac{N_2}{N_1} = \left( \frac{D_1 + t}{D_2 + t} \right) \left( \frac{100 - S}{100} \right) \quad \text{or} \quad \frac{150}{80} = \left( \frac{600 + 5}{D_2 + 5} \right) \left( \frac{100 - 4}{100} \right) \]
\[ D_2 = 304.8 \text{mm} \]

iv. \[
\frac{N_2}{N_1} = \left( \frac{D_1 + t}{D_2 + t} \right) \left( \frac{100 - S}{100} \right)
\]

Where \( S = S_1 + S_2 - 0.01S_1 S_2 \)

\[ = 2 + 2 - 0.01 \times 2 \times 2 \]

\[ = 3.96 \]

\[
\frac{150}{80} = \left( \frac{600 + 5}{D_2 + 5} \right) \left( \frac{100 - 3.96}{100} \right)
\]

\[ D_2 = 304.9 \text{ mm} \]

RATIO OF FRICTION TENSIONS

1. Flat belt

Let \( T_1 = \) tension on tight side

\( T_2 = \) tension on slack side

\( \theta = \) angle of lap of the belt over the pulley

\( \mu = \) coefficient of friction between the belt and the pulley

Consider a short length of belt subtending an angle \( \delta \theta \) at the center of the pulley

Let \( R = \) normal (radial) reaction between the element length of belt and the pulley

\( T = \) Tension on slack side of the element

\( \delta T = \) increase in tension on tight side than that on slack side
\( T + \delta T \) = tension on tight side of the element

Tensions \( T \) and \( (T + \delta T) \) act in directions perpendicular to the radii drawn at the ends of the element. The friction force \( \mu R \) will act tangentially to the pulley rim resisting the slipping of the elementary belt on the pulley.

\[ \mu R + T \cos \frac{\delta \theta}{2} - (T + \delta T)\cos \frac{\delta \theta}{2} = 0 \]

As \( \delta \theta \) is small,

\[ \cos \frac{\delta \theta}{2} \approx 1 \]

\[ \therefore \mu R + T - T - \delta T = 0 \quad \text{or} \quad \delta T = \mu R \quad \text{(i)} \]

Resolving the forces in the radial direction,
\[ R - T \sin \frac{\delta \theta}{2} - (T + \delta T) \sin \frac{\delta \theta}{2} = 0 \]

As \( \delta \theta \) is small, \( \sin \frac{\delta \theta}{2} \approx \frac{\delta \theta}{2} \)

\[ \therefore R - T \frac{\delta \theta}{2} - T \frac{\delta \theta}{2} - \frac{\delta T \delta \theta}{2} = 0 \]

Neglecting product of two small quantities,

\[ R = T \delta \theta \] \hspace{1cm} (ii)

From (i) and (ii), \( \delta T = \mu T \delta \theta \) \hspace{1cm} or \( \frac{\delta T}{T} = \mu \delta \theta \)

Integrating between proper limits,

\[ \int_{T_2}^{T_1} \frac{dT}{T} = \int_0^\theta \mu d\theta \]

Or \( \log_e \frac{T_1}{T_2} = \mu \theta \)

Or \( \frac{T_1}{T_2} = e^{\mu \theta} \)

It is to be noted that the above relation is valid only when the belt is on the point of slipping on the pulleys.

**POWER TRANSMITTED**

Let \( T_1 \) = tension on the tight side

\( T_2 \) = tension on the slack side

\( v \) = linear velocity of the belt

\( p \) = power transmitted

Then,
P = net force x distance moved/second

= (T_1 - T_2) \times v

This relation gives the power transmitted irrespective of the fact whether the belt is on the point of slipping or not. If it is, the relationship between T_1 and T_2 for a flat belt is given by

\[
\frac{T_1}{T_2} = e^{\mu \theta}.
\]

If it is not, no particular relation is available to calculate T_1 and T_2.

**CENTRIFUGAL EFFECT ON BELTS**

![Fig 3.19](image)

While in motion, as a belt passes over a pulley, the centrifugal effect due to its own weight tends to lift the belt from the pulley. Owing to symmetry, the centrifugal force produces equal tensions on the two sides of the belt, i.e. on the tight sides and the slack sides.

Consider a short element of belt (fig 3.19)

Let \( m = \) mass per unit length of belt

\[ T_c = \text{centrifugal tension on tight and slack sides of element} \]
\[ F_c = \text{centrifugal force on the element} \]
\[ r = \text{radius of the pulley} \]
\[ v = \text{velocity of the belt} \]
\[ \delta \theta = \text{angle of lap of the element over the pulley} \]
\[ F_c = \text{mass of element} \times \text{acceleration} \]
\[ = (\text{length of element} \times \text{mass per unit length}) \times \text{acceleration} \]
\[ = (r \delta \theta \times m) \times \frac{v^2}{r} \]
\[ = mv^2 \delta \theta \quad \text{(i)} \]

Also,
\[ F_c = 2T_c \sin \frac{\delta \theta}{2} \]

As \( \delta \theta \) is small,
\[ \sin \frac{\delta \theta}{2} \approx \frac{\delta \theta}{2} \]

\[ F_c = 2T_c \frac{\delta \theta}{2} \]
\[ = T_c \delta \theta \quad \text{(ii)} \]

From (i) and (ii),
\[ T_c \delta \theta = mv^2 \delta \theta \]

Or
\[ T_c = mv^2 \]

Thus centrifugal tension is independent of the tight and slack side tensions and depends only on the velocity of the belt over the pulley.
Also,

\[
\text{Centrifugal stress in the belt} = \frac{\text{centrifugal tension}}{\text{area of cross-section of belt}}
\]

\[
= \frac{T_c}{a}
\]

Total tension on tight side = friction tension + centrifugal tension

\[
T = T_1 + T_c
\]

Total tension of slack side = \(T_2 + T_c\)

It can be shown that the power transmitted is reduced if centrifugal effect is considered for a given value of the total tight side tension \(T\).

(a) Centrifugal tension considered

Friction tension on tight side = \(T - T_c = T_1\)

Let \(T_2\) be the friction tension on the slack side.

Then \[
\frac{T_1}{T_2} = e^{\mu \theta} = k, \text{ a constant}
\]

Or \[
T_2 = \frac{T_1}{k}
\]

And power, \(P = (T_1 - T_2)v = (T_1 - \frac{T_1}{k})v = T_1 (1 - \frac{1}{k})v\)

(b) Centrifugal tension Neglected

Friction tension on tight side = \(T\)

Let \(T_2'\) be the friction tension on slack side.

\[
\frac{T}{T_2'} = e^{\mu \theta} = k \quad \text{Or} \quad T_2' = \frac{T}{k}
\]

Power, \(P = (T - T_2')v = (T - \frac{T}{k})v = T (1 - \frac{1}{k})v\)

As \(T_1\) is lesser than \(T\), power transmitted is less when centrifugal force is taken into account.
MAXIMUM POWER TRANSMITTED BY BELT

If it is desired that a belt transmits maximum possible power, two conditions must be fulfilled simultaneously.

They are

1. Larger tension must reach the maximum permissible value for the belt
2. The belt should be on the point of slipping, i.e. maximum frictional force is developed in the belt.

Now,

\[ P = (T_1 - T_2)v = T_1 \left(1 - \frac{T_2}{T_1}\right) v = T_1 \left(1 - \frac{1}{e^{μB}}\right) v = T_1 \ k \ v \]

Where \[ k = 1 - \frac{1}{e^{μB}} = \text{constant} \]

Or \[ P = (T - T_c) \ k \ v \]

\[ = k \ T \ v - k \ m \ v^2 \]

\[ = k \ T \ v - k \ m \ v^3 \]

The maximum tension \( T \) in the belt should not exceed the permissible limit. Hence treating \( T \) as constant and differentiating the power with respect to \( v \) and equating the same equal to zero.

\[ \frac{dP}{dv} = k \ T - 3 \ k \ m \ v^2 = 0 \]

Or \[ T = 3 \ m \ v^2 = 3T_c \]

Or \[ T_c = \frac{T}{3} \]
Therefore, for maximum power to be transmitted centrifugal tension in the belt must be equal to one-third of the maximum allowable belt tension and the belt should be on the point of slipping.

Also,

\[ T_1 = T - T_c = T - \frac{T}{3} = \frac{2}{3} T \]

And
\[ v_{\text{max}} = \frac{T}{\sqrt{3m}} \]

**Problems on flat belt drive**

**Problem 1** A shaft running at 200 r.p.m is to drive a parallel shaft at 300 r.p.m. The pulley on the driving shaft is 60 cm diameter. Calculate the diameter of the pulley on the driven shaft:

(1) Neglecting belt thickness.

(2) Taking belt thickness into account, which is 5 mm thick.

(3) Assuming in the latter case a total slip of 4%.

**Sol.** Given:

\[ N_1 = 200 \text{ r.p.m.}, \quad N_2 = 300 \text{ r.p.m}, \]

And

\[ d_1 = 60 \text{ cm}, \quad t = 5 \text{ mm} = 0.5 \text{ cm} \]

Total slip \( s = 4\% \).

(1) Neglecting belt thickness.

Let \( d_2 = \) Dia. Of the pulley on driven shaft.

Using equation (7.1), we get

\[ \frac{N_2}{N_1} = \frac{d_1}{d_2} \]

\[ d_2 = \frac{N_1}{N_2} \times d_1 = \frac{200}{300} \times 60 = 40 \text{ cm. Ans} \]
(2) Taking belt thickness into account only.

Using equation, we get \[ \frac{N_2}{N_1} = \left( \frac{d_1 + t}{d_2 + t} \right) \]

Or \[ (d_2 + t) = (d_1 + t) \times \frac{N_1}{N_2} \]

\[ = (60 + 0.5) \times \frac{200}{300} = 60.5 \times \frac{2}{3} \]

\[ d_2 = 60.5 \times \frac{2}{3} - 0.5 = \frac{121}{3} - 0.5 \]

\[ = 40.33 - 0.5 = 39.83 \text{ cm. Ans.} \]

(3) Considering belt thickness and total slip.

Using equation, we get \[ \frac{N_2}{N_1} = \left( \frac{d_1 + t}{d_2 + t} \right) \left( 1 - \frac{s}{100} \right) \]

\[ \frac{300}{200} = \left( \frac{60+0.5}{d_2+0.5} \right) \left( 1 - \frac{4}{100} \right) \]

\[ (d_2 + 0.5) = 60.5 \times 0.96 \times \frac{200}{300} = 38.72 \]

\[ d_2 = 38.72 - 0.5 = 38.22 \text{ cm Ans.} \]

**Problem 2** The power is transmitted from a pulley 1 m diameter running at 200 r.p.m. to a pulley 2.5 m diameter by means of a belt. Find the speed lost by the driven pulley as a result of the creep, if the stress on the tight and slack side of the belt is 1.44 N/mm² and 0.49 N/mm² respectively. The young's modulus for the material of the belt is 100 N/mm².

Sol. Given:

Dia. Of driver pulley, \[ d_1 = 1 \text{ m} \]

Speed of driver pulley, \[ N_1 = 200 \text{ r.p.m} \]

Dia. Of driven pulley, \[ d_2 = 2.5 \text{ m} \]

Stress on tight side, \[ f_t = 1.44 \text{ N/mm}^2 \]

Stress on slack side, \[ f_2 = 0.49 \text{ N/mm}^2 \]
Young’s modulus, \( E = 100 \text{ N/mm}^2 \)

Let \( N_2 = \text{speed of driven pulley} \).

Using equation, we get
\[
\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{f^2}}{E + \sqrt{f^2}}
\]

Or
\[
\frac{N_2}{200} = \frac{1}{2.5} \left[ \frac{100 + \sqrt{0.49}}{100 + \sqrt{1.44}} \right] = \frac{1}{2.5} \left[ \frac{100.7}{101.12} \right]
\]

\[ N_2 = \frac{200}{2.5} \times \frac{100.7}{101.12} = 79.67 \text{ rpm} \]

If the creep is neglected, then from equation, we get
\[
\frac{N_2}{N_1} = \frac{d_1}{d_2}
\]

\[ N_2 = \frac{d_1}{d_2} \times N_1 = \frac{1}{2.5} \times 200 = 80 \text{ r.p.m} \]

\[ \therefore \text{Speed lost by driven pulley due to creep} = 80 - 79.67 = 0.33 \text{ rpm}. \]

Ans.

**Problem 3** An open belt drive connects two pulleys 120 cm and 50 cm diameters, on parallel shafts 4 m apart. The maximum tension in the belt is 1855.3 N. The coefficient of friction is 0.3. The driver pulley of diameter 120 cm runs at 200 r.p.m calculates:

1. The power transmitted, and

2. Torque on each of the two shafts.

Sol. Given:

Dia. Of larger pulley, \( d_1 = 120 \text{ cm} = 1.20 \text{ m} \)

Radius of larger pulley, \( r_1 = \frac{120}{2} = 60 \text{ cm} = 0.6 \text{ m} \)

Dia. Of smaller pulley, \( d_2 = 50 \text{ cm} = 0.50 \text{ m} \)

Radius of smaller pulley, \( r_2 = 25 \text{ cm} = 0.25 \text{ m} \)

Distance between the shafts, \( x = 4 \text{ m} \)
Max. Tension, \( T_1 = 1855.3 \text{ n} \)

Coefficient of friction, \( \mu = 0.3 \)

Speed of driver pulley, \( N_1 = 200 \text{ r.p.m} \)

We know that velocity of belt is given by,

\[
v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.56 \text{ m/s}
\]

Let us now calculate the angle of contact \( (\theta) \). For an open belt drive angle of contact is given by equation (7.11) as

\[
\theta = 180 - 2 \alpha \quad (1)
\]

Where \( \alpha \) is given by equation as

\[
\sin \alpha = \frac{r_1 - r_2}{x} = \frac{0.6 - 0.25}{4} = 0.0875
\]

\[
\therefore \quad \alpha = \sin^{-1} 0.0875 = 5.02^\circ
\]

Substituting this value of \( \alpha \) in equation (1), we get

\[
\theta = 180 - 2 \times 5.02 = 169.96^\circ
\]

\[
= 169.96 \times \frac{\pi}{180} \text{ radians} = 2.967 \text{ radians.}
\]

Let \( T_2 = \) Tension on slack side of the belt.

Now using equation,

\[
\frac{T_1}{T_2} = e^{\mu \theta}
\]

Or

\[
\frac{1855.3}{T_2} = e^{0.3 \times 2.967} = e^{0.8901} = 2.435
\]

\[
\therefore \quad T_2 = \frac{1855.3}{2.435} = 761.8 \text{ N}
\]

(1) Power transmitted. Using equation,

\[
P = \frac{(T_1 - T_2) \times V}{1000}
\]
\[
\frac{(1855.3 - 761.8) \times 12.56}{1000} = 13.73 \text{ kW. Ans}
\]

(2) Torque on each of the two shafts. Torque exerted on the driving shaft is given by equation (\(\cdot\)).

\[
\text{Torque} = (T_1 - T_2) \times r_1
\]

\[
= (1855.3 - 761.8) \times 0.6 = 656.1 \text{ Nm. Ans.}
\]

Torque exerted on the driven shaft is given by equation

\[
\text{Torque} = (T_1 - T_2) \times r_2
\]

\[
= (1855.3 - 761.8) \times 0.25 = 273.4 \text{ Nm. Ans.}
\]

Problem 4. A belt of density 1 gm/ cm\(^3\) has a maximum permissible stress of 250 N/cm\(^2\). Determine the maximum power that can be transmitted by a belt of 20 cm x 1.2 cm if the ratio of the tight side to slack side tension is 2.

Sol. Given:

Density of belt, \(\rho = 1 \text{ gm/cm}^3 = \frac{1}{1000} \text{ kg/cm}^3\)

Max. Permissible stress, \(f = 250 \text{ N/cm}^2\)

Width of belt, \(b = 20 \text{ cm}\)

Thickness of belt, \(t = 1.2 \text{ cm}\)

Ratio of tensions, \(\frac{T_1}{T_2} = 2.0\)

Let us first find the mass of 1 m length of the belt and also the maximum tension in the belt.

Let \(m = \text{mass of one meter length of belt}\)

\[
= (\text{density}) \times \text{volume of belt of one meter length}
\]

\[
= \frac{1}{1000} \times (b \times t \times 100)
\]

\[
= \frac{1}{1000} \times 20 \times 1.2 \times 100 \text{ kg} = 2.40 \text{ kg}
\]
Using equation,

\[ T_m = \text{maximum tension} = (\text{max. stress}) \times \text{Area of cross- section of belt} \]
\[ = 250 \times b \times t = 250 \times 20 \times 1.2 = 6000 \text{ N.} \]

Now for maximum power transmitted, the velocity of the belt is given by equation as

\[ v = \sqrt{\frac{T_m}{3m}} = \sqrt{\frac{6000}{3 \times 2.4}} \text{ m/s} = 28.86 \text{ m/s}. \]

Now power transmitted is given by equation,

\[ P = \frac{(T_1 - T_2) \times V}{1000} \quad (1) \]

Let us find the values of \( T_1 \) and \( T_2 \).

We know that \( T_m = T_1 + T_c \) \quad (2)

Where \( T_c \) = centrifugal tension, and

\( T_1 \) = tight side tension.

But for maximum power transmission, \( T_c = \frac{1}{3} T_m \)

Substituting this value in equation (2), we get

\[ T_m = T_1 + \frac{1}{3} T_m \]

Or \( T_1 = T_m - \frac{1}{3} T_m = \frac{2}{3} T_m = \frac{2}{3} \times 6000 = 4000 \text{ N} \)

But \( \frac{T_1}{T_2} = 2.0 \) (given)

\[ T_2 = \frac{T_1}{2} = \frac{4000}{2} = 2000 \text{ N} \]

Substituting the values of \( T_1 \), \( T_2 \) and \( v \) in equation (1), we get

\[ P = \frac{(4000 - 2000) \times 28.86}{1000} = 57.18 \text{ kW.} \quad \text{Ans.} \]