Introduction and Concepts

Here we shall learn basic concepts, Defn of Manufacturing process,Product example,General classification of Mnf process, What is Mechanical working, Deformation,Stress, Ductile and Brittle metals, Classification of Mech Working-process,Concept of RCT, Hot ,Cold and Warm working.

Manufacturing Process contain two words:
Manufacturing and Process.
What is Manufacturing? ,,,,,,,,,,,,,,,
Manufacturing is the conversion of raw materials into a useful item called as Product.
Product is the desired or wanted material which has shape, size, colour, and moreover has Function to perform.
What is a Process?
Process is the step by operations involved in the conversion of raw material.
Ex., Coffee is the product → It is used as a drink. It is to satisfy the customer. This is its function.
Coffee Powder,Sugar,Water,Milk are the raw materials in addition a vessel and fuel is used for the purpose.
The raw materials are converted to the drink called as coffee. This is Manufacturing.
The steps involved water is taken in a vessel and heated to boiling using a heat source, then coffee powder is added and after some time it is filtered to get the decoction. To this sugar and milk are added to get the coffee. The steps involved above is the Process.
Similarly Pen is a product, Shirt is a product ….
Now we shall try to understand what is a product. Let us take the Pen… It has shape, size, colour and a function to perform ie.,writing on a paper leaving marks.

Manufacturing Processes can be classified as i) Casting ii) Welding iii) Machining iv) Mechanical working v) Powder Metallurgy vi) Plastic Technology etc.,

Here we shall concentrate on one of the important process ie., Mechanical working Process.
In this process the raw material is converted to a given shape by the application of external force. The metal is subjected to stress.
Stress: It is defined as the ratio of force to area of cross section of the material on which the force is acting.

The raw material undergoes change in the shape and this is referred to as Deformation.

Deformation: It is a process of changing the shape and size of the material under the influence of external force or stress.
Deformation can be Elastic or Plastic in nature.
Elastic deformation is a temporary one ie.,the metal undergoes change in shape and size under the influence of external load and regains the same once the load is removed.
Plastic deformation is a permanent one ie.,the metal undergoes change in shape and size under the influence of external load and cannot regain the original state.
Basically a Metal is used for mechanical working process. All metals which are ductile are mechanically worked.
A Ductile metal is one which undergoes increase or decrease in length when external force is applied without breaking. We have sheet or wire which are obtained due to ductility property.ex., mild steel.
There are brittle metals also. A brittle metal is one which will not undergo extension or decrease in length under the influence of external force but fracture or fail.ex., cast iron

***Classification of Metal Working Processes**

1.General classification

i. Rolling
ii. Forging
iii. Extrusion
iv. Wire Drawing
iv. Sheet Metal Forming

2.Based on Temperature of Working

i.Hot Working
ii.Cold Working
iii.Warm Working

3. Based on the applied stress

i.Direct Compressive Stress
ii.Indirect Compressive Stress
iii.Tensile Stress
iv.Bending Stress
v)Shear Stress

Product: let us try to understand the meaning with an example.
Consider a Liter Can……It is used to measure the volume of grains or liquid or powder.
It is having shape ..Cylindrical.
It is having size …. Diameter and height.
It is having colour… Based on the material.
It has function to perform.. To measure volume of materials.
Similarly one can identify many many products in our daily life and understand.

We shall know about Classification of Metal Working based on temperature first.

Hot working: It is defined as the mechanical working of metal at an elevated (higher) temperature. This temperature is referred to RCT(Re Crystallization Temperature).

Cold Working: It is defined as the mechanical working of metal below RCT.

Warm Working: It is defined as the mechanical working of metal at a temperature between that of Hot working and Cold Working.

What is RCT? It is the temperature at which all metallic materials undergo fragmentation of the grains followed by nucleation and growth, under the influence of the external force.

What is the value of RCT? The value of RCT has been arrived at by conducting large number of studies. Its value lies between 0.4-0.5 times the Melting point of the metal. It is a function of the nature of the metal.

Ex., For Aluminium the melting point is 660^{o}C. Hence RCT is 330^{o}C.

Books for Reference

1. Dieter,Mechanical Metallurgy ,

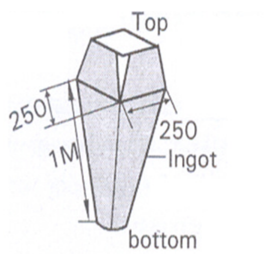
2.Dr.K.Radhakrishna,Manufacturing ProcessIII,IEdition August 2009, Sapna Book House,

Unit 1(Class2) Introduction and Concepts

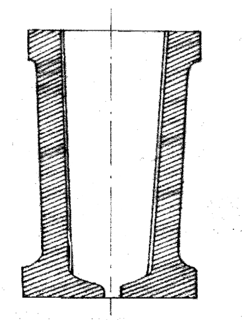
Here we shall study Types of Stresses used for Deformation, Different Mechanical Working Process, Concept of Ingot , How produced, Steps involved in Mechanicxal Working Process, Bloom, Billet,Bar,Slab,Plate,Sheet,Foil,Structural Shapes.

In mechanical working process the raw material is the metal in some form which is subjected to external force.

The raw material is obtained by pouring molten metal into a metallic mould and after solidification the solid metal is taken out. This will have a given cross section such as square or rectangle or circular etc,. and of a given length. This is referred to as Ingot.



Typical Ingot Casting.



Typical Ingot Mould

Ingot is the starting raw metal for all metal working process.

Molten metal from the furnace is taken and poured into metal moulds and allowed to cool or solidify. The cooled solid metal mass is then taken out of the mould. This solid metal is referred to as Ingot.

This Ingot is later on converted to other forms by mechanical working.

Let us see the conversion through

Rolling and Forging

Rolling: It is a process wherein the ingot is passed between the gaps of two rotating rolls to get deformation. It is similar to sugar cane juice making …. Here there will be two rotating cylindrical rolls between which sugar cane is passed and crushed to get the juice. The sugar cane is getting crushed.

Forging: Here the metal is pressed between two hard surfaces to get a reduced section.

It is similar to pressing a model clay ball between two fingers or palms. The clay gets crushed to smaller thickness.

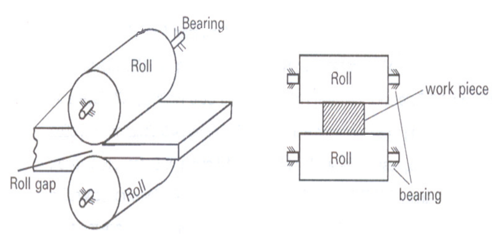
a) Rolling Route

*Molten metal → poured into metal moulds and cooled →Ingot (≈250x250mm)
*Ingot→ rolled→Blooms(≈200x200mm)
*Bloom→rolled→Billets(≈150x150mm)
*Billet→rolled→Bars/Rods(≈40x40mm)
*Bar→drawn into Wires(<5mm dia.)
*Billet→rolled→Slabs(t<b)
*Billet→rolled Structural shapes I,U,L,V etc.
*Slabs→Plates(t<<b and t>>4mm)
*Plate→Sheets(t<4mm)→Foils(microns)
b)Forging Route
*Ingot→Open die→Blooms→Billets→Bars
*Ingot→Closed die→Shaped objects like crank shaft, spanner, connecting rod etc.,
As said earlier one has to apply external force or load or stress to bring about the deformation in the metal.
We shall learn what types of stresses are present in each of the mechanical working process.

***Rolling --- Direct Compressive Stress**

Here Plastic deformation takes place in metal when passed through a set of two rotating cylindrical rolls.

As the hot metal is passed through the gaps of two rotating rolls the metal is dragged inside and the compression of the metal takes place and the cross section is reduced. During the process there is a contact point at the start and an exit point at the other end. During these points the metal is in complete contact with the rolls and the metal experiences direct compressive stress as shown in the figure.



Typical Rolling process

***Forging ----- Direct Compressive Stress**

Here a simple open type forging is taken for discussion. There will be two flat hard surfaces called as dies. The gap between the dies can be adjusted and the metal is placed between the two dies and load is applied. When the dies press against the metal it gets squeezed and the cross section is reduced. The metal will experience direct compressive stress. Fig.shows this.

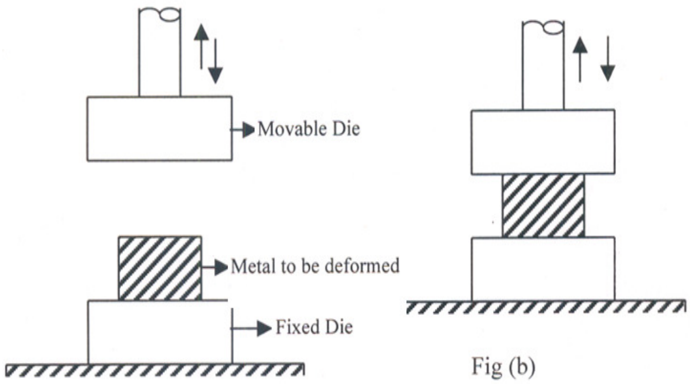
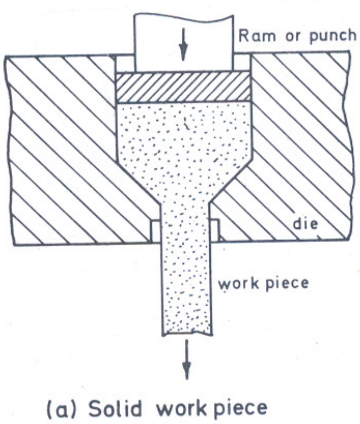


Fig (a)

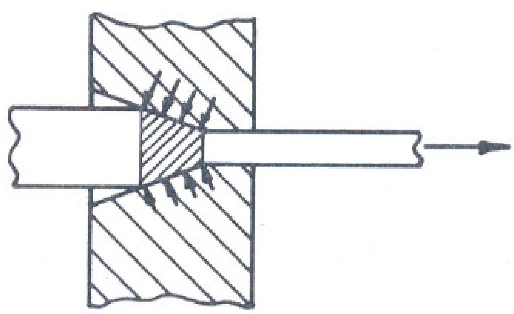
Fig (b)

***Extrusion and Wire Drawing Indirect Compression**

Here a conical die is used for deforming the metal. The die is a converging one.
In Extrusion the metal is pushed from one end towards the conical opening and the cross section is reduced.
In Wire drawing the metal is pulled from the front end of the conical end and the cross section is reduced.
In both cases the metal is subjected to indirect compressive stress.Fig. shows this.



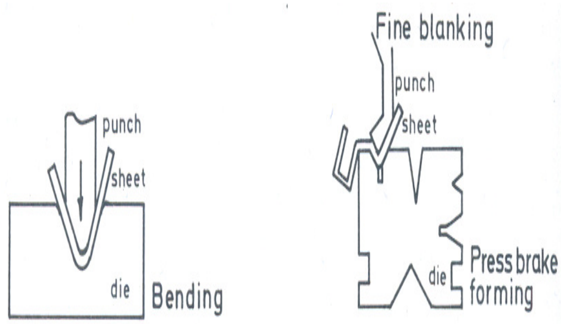
Extrusion



Wire drawing

***Bending of Sheet ----Bending Stress**

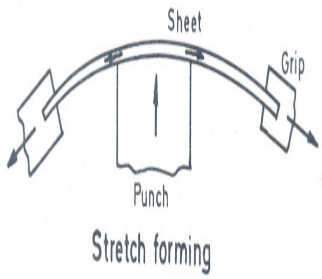
Two dies with matching contours are used. The metal in the form of sheet is placed on the bottom die and the top die is brought in contact with the metal and force is applied and the metal takes the contour of the dies. The metal is subjected to differential stresses on either side of the sheet resulting in bending stresses in the metal. Fig. shows this.



Processes making use of bending stresses

***Stretching of Sheet ---Tensile Stress**

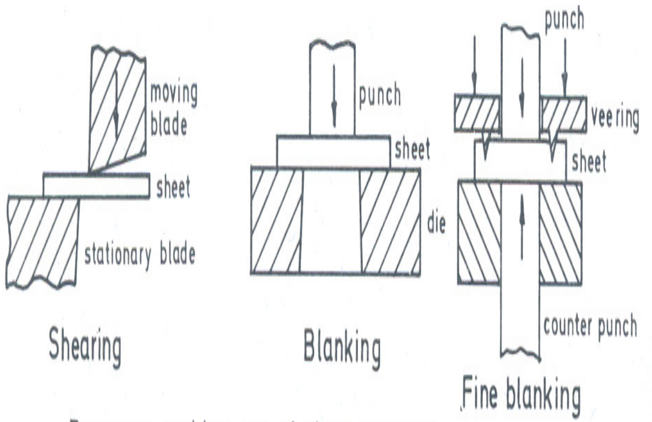
Here sheet metal is held between grippers and the sheet is made to press against a contoured mandrel. The process is repeated to get a permanent change in the shape. Normally this is used to deform large surfaces. The sheet is being stretched and hence experiences tensile stresses. Fig. shows this.



Process making use of tensile stress

***Shearing of Sheet ----Shear Stress**

This is used to cut the sheet into two pieces. Here two sharp edges called as shears are used(similar to scissors). The sheet metal is kept between the two shears and force is applied. The metal pieces get separated. Fig. shows this.



Processes making use of shear stresses

Unit 1 (Class3) Introduction and Concepts

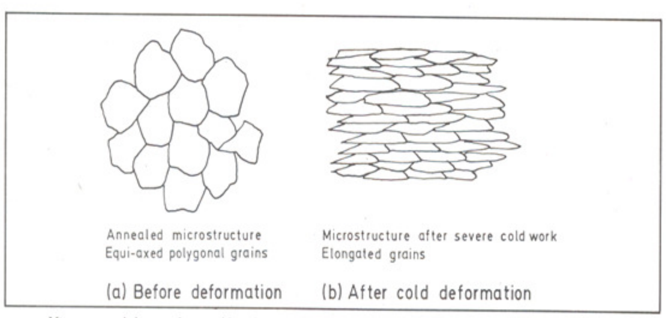
What is a Cast Product?

It is a product obtained by just pouring molten metal into the mould and allowing it to solidify to the room temperature. It will have the final size and shape.

Engine block ,Piston etc.,

What is a wrought Product?

It is a product obtained by subjecting the hot ingot to mechanical working process and get a variety of products.
Ex., spanner, screw driver, connecting rod, etc.,
Characteristics of a Wrought Product
*Grains are oriented in a particular direction.
*The metal will show flow lines which are due to the presence of inclusions present between the metal layers.
*The Metal will shows higher properties in the direction of metal flow.*The defects get welded due to mechanical working.



Microstructural changes due to cold working.

Difference between Cast and Wrought product.

Cast Product	Wrought Product
1.It is obtained by conversion of liquid metal to solid state to get the required shape of the component in one step. The cast product may undergo machining operation.	1. It is obtained by subjecting the metal to external load or mechanical working to get the shape. It may be subjected to further operation.

2. The cast product will have uniform properties. The product is Isotropic in nature.	2. The wrought product will have directional properties. Properties are Enhanced. The Product is anisotropic in nature. ie.,properties are different in different directions.
3. The product will have uniform grain structure.	3. The product will have directional properties and the grain structure will be oriented. Grains get altered.
4. The product will have small amount of porosity which cannot be eliminated completely.	4. Due to mechanical working the porosity level is almost zero.
5. Cast product will have any shape size and complexity. Small to very huge components can be produced easily.	5. Wrought products can also be produced with large size and moderately complex shapes with difficulty.
6. Even brittle metals can be cast easily.	6. Brittle metals cannot be mechanically worked.
7. The process does not need additional equipments. Only Regular maintenance of the equipment is required.	7. Needs additional equipments for mechanical working.

Difference between Hot Working and Cold Working

Hot Working	Cold Working
1. The metal is subjected to mechanical working above RCT	1. The metal is subjected to mechanical working below RCT.
2. Uniform fine equiaxed grains are o Properties are uniform in any direction.	2. Elongated columnar grains are fo Properties are higher in a particular direction.
3.Properties are uniform in any direction.	3.The properties are higher in a particular direction than the other. 4. The component will be anisotropic in

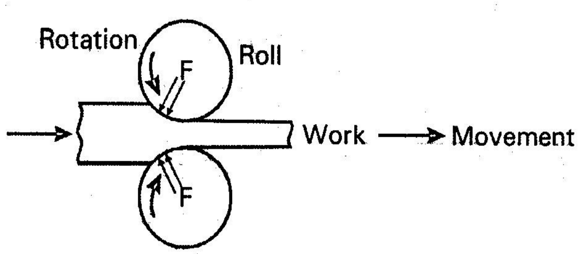
4. The components will have Isotropic properties.	nature.
5. Energy required for deformation is less.	5. Energy required for deformation is more.
6. No strain hardening takes place in the metal.	6. Strain hardening takes place in the metal.
7. Large components can be hot worked easily.	7. Only small component s can be cold worked.
8. Surface oxidation occurs, scaling will be present and surface finish is poor. 9. Pores are minimized and inclusion gets redistributed.	8. Surface oxidation is less, scale formation is also less and surface finish is good. 9. Pores and inclusions cannot be taken care of to a great extent.
10. Capacity of the equipment is less.	10. The capacity of the equipment required for cold working is less.
11. Needs extra equipment for heating of the metal.	11. Does not need extra equipment as in hot working.
12. Handling of hot metal is difficult.	12. Handling of metal is not very difficult.

Classification of Metal Working Process based on Stresses

1. Direct Compressive Stress

a) Rolling and b) Forging

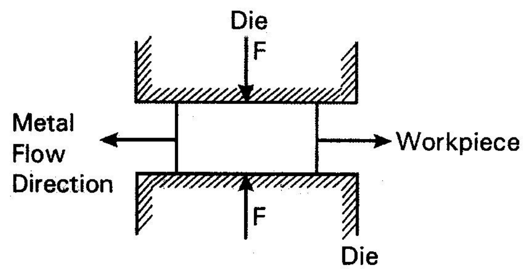
Rolling



Rolling

It consists of two rotating rolls with a gap between which the metal is passed through and the metal gets reduced in its cross section. During the process the metal experiences direct compression as shown in the figure.

Forging



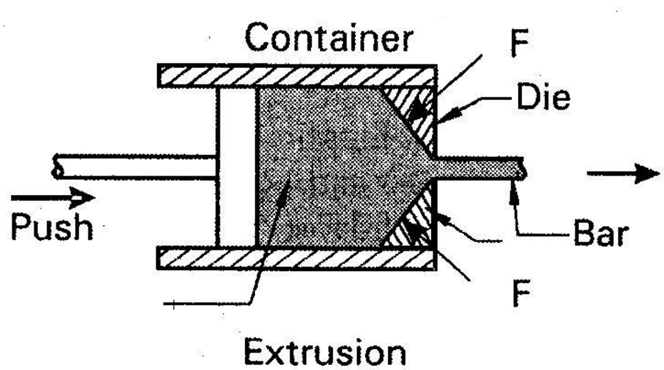
Open die Forging

Here the metal is placed between the gap of two hard surfaces called as dies and are made to come closer and make contact with the metal. On applying the load the metal is compressed and it experiences direct compression.

2. Indirect Compression

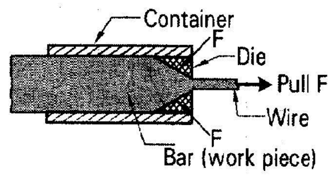
a. Extrusion and b) Wire Drawing

Extrusion



It consists of a cylindrical container in which the metal is kept and pushed out of a conical end called as die. Reduction in the cross section takes place. The metal experiences indirect compression near the die walls.

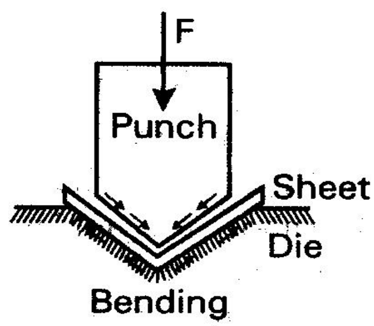
Wire drawing



Here also a cylindrical container with a conical end will be there and the metal is being pulled out of the die. The metal cross section is smaller than in extrusion.

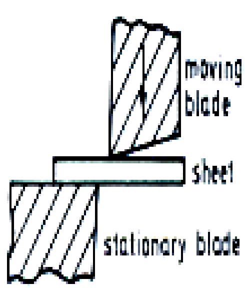
3. Bending Stress

Bending of plates and sheets



Here two dies are used with a given contour as shown in the fig. A sheet is placed on the bottom die and the upper die is made top force against the sheet and the metal undergoes bending due to tensile and compressive forces acting on it at top and bottom surfaces. The metal takes the form of the contour and bends.

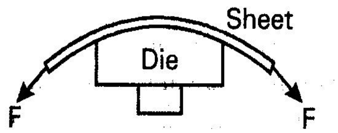
4. Shear Stress



Shearing

Here a sheet is cut between two sharp edges called as shear blades. The metal is placed on one of the blades and the other blade is made to force against it. The metal undergoes shear as shown in the fig. Pure shear stress will be acting on the metal.

5. Tensile Stress



Stretch forming

A sheet metal is held between grippers and is pulled against a contoured die as shown in the fig. The sheet takes the form of the die due to tensile forces acting on it. The required contour is obtained.

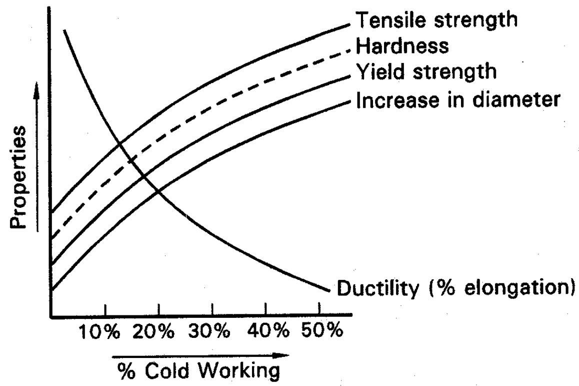
UNIT 1 (Class4) Introduction and Concepts

We shall learn about advantages and limitation of MW process concept of cold working and strain hardening, cold working Vs properties, different Stress –Strain curves, concept of stress, strain and deformation.

Advantages of metal working process
* Product with consistent high quality can be manufactured.
* Defects such as porosity and discontinuities are minimized.
* Inclusions get distributed evenly throughout the product.
* Grains are oriented in a particular direction and directional properties are obtained.
* In hot working the grains will be uniform and the properties are also uniform.
* In cold working the properties are enhanced due to strain hardening effect.
* Large tonnage can be easily produced.
*The process can be easily mechanized.

Limitations of Mechanical working process
*The product becomes highly anisotropic in nature.
*Final product has to be obtained after machining of the wrought product except in the case of structural components.
*Needs additional equipment and machinery for metal working process. Hence, initial investment is high.
*Maintenance cost is high.
*More safety precautions are to be exercised as hot metal and additional equipments are used.

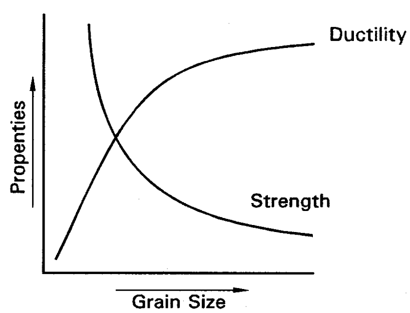
Concept of cold working
Consider a cylindrical metal piece with a known height, H and diameter, D. Let us subject the piece to compressive load at room temperature. We shall take that the height is reduced by 10%,20%,30% ….. etc., Each of these reduction in height represents %cold working. For each of these the diameter the Tensile strength, hardness, yield strength, %elongation were measured. It is seen that the %elongation decreases with increase in %cold working whereas other properties UTS, YS, Hardness increases and the diameter of the specimen also increases as shown in the figure.
Similarly the specimen can be subjected to tensile load also.
The changes that take place in the material due to cold working is an important aspect which needs to be born in mind while designing various steps in MW process.



Effect of cold working on properties

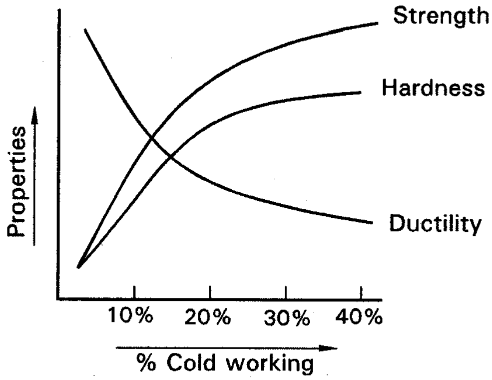
Effect of Mechanical Working on the properties of the Metal

Due to working of the metal there will be changes in the grain structure. The grains may be elongated in one direction from its equiaxed shape. The pores are reduced and the inclusions are fragmented and distributed evenly in the metal. In hot working the coarse equiaxed grains will become fine equiaxed. The changes obtained in cold working is appreciable.
The behaviour of the metal with changes in grain size is shown in the figure. As grain size becomes coarse the strength property comes down and ductility increases.



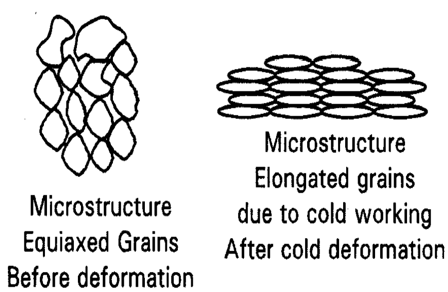
Variation of properties with respect to grain size

As the percentage of cold working increases the material becomes strain hardened, the hardness and strength properties are increased but the ductility property decreases as shown in the figure.



Variation of properties as a function of % cold working

It can be summarized as follows. Equiaxed grains will give uniform properties in all directions. Deformed grains show higher strength properties in the elongated direction.



Effect of cold working on grain structure

Concept of strain hardening

Straining of the metal/alloy occurs when subjected to cold working process. The metal will show more and more resistance for the external load as the cold working is Continued. At some it may become very difficult to deform the metal. This phenomenon is referred to as strain hardening effect. This can be explained in simple terms as given below.

All metals have atoms arranged in a repetitive manner in three dimensions referred to as crystalline structure. The structure is associated with imperfections in the form of dislocations. These dislocations starts moving towards the grain boundary region under the influence of external load. The dislocations get piled up near the grain boundaries. The density of dislocations increases due to Frank Reed source and may reach a value as high as 10^8-$10^{12}/cm^2$. Since, dislocations pile up near the grain boundary the density increases and the mean free path for the movement of dislocations decreases. The metal offers more resistance to external force. The metal will realize higher strength and this goes on building up till all the dislocations are brought near the grain boundary. Then annihilation of like and unlike dislocations takes place. The net existing dislocations will then become effective. During this period the load required for

deformation increases. This phenomenon is referred to as"Strain Hardening". If the cold working stress exceeds this range the metal will fracture.
To take care of this the metal is subjected to annealing before further working.

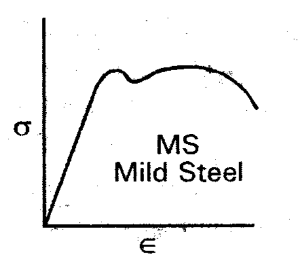
In Mechanical working of metals, the metal is subjected to external load and is deformed plastically. The given shape is obtained and is retained even after the removal of the load. The metal is subjected to stress and is strained. Hence, to understand the different mechanical working process, it is necessary to understand the stress strain relationship of metals, types of stress and strains, deformation process, theories used for the prediction of plastic deformation etc.,

For easy mechanical working of metals the nature of stress strain curve needs to be **reviewed.** The factors associated with stress strain needs to be studied.

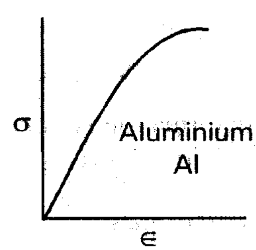
Different stress- strain curves

Some typical stress strain curves are shown below

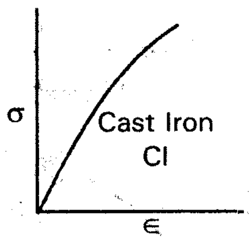
Different stress strain curves
There exists relationship between stress and strain for all materials and it is very useful information for a design engineer and metallurgist alike. It clearly exhibits the behavior of the material. In order to understand the same we shall now look into stress strain curves of various materials. A study of these will help us in understanding the mechanical working process in a better way. The following figures represents pictorially the features of stress **and strain** behavior.



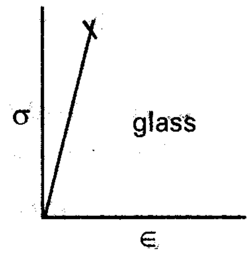
a) Highly ductile material



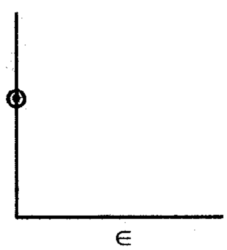
b) Mildly ductile material



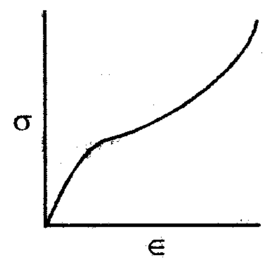
c) Brittle material



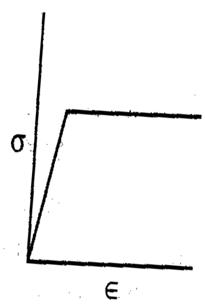
d) Highly brittle material



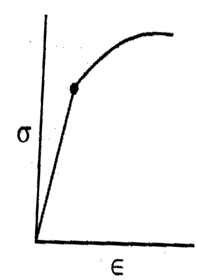
e) Highly Rigid material



f) Polymer material



g) Linear elastic - perfectly plastic material



g) Linearly elastic - Non linearly plastic material

Typical stress strain curves for easy deformation

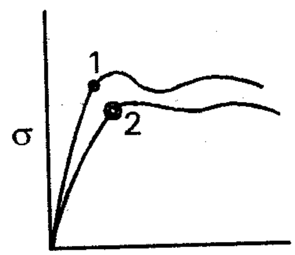
In Mechanical working of metals it is important to know that efforts are to be made to make the metal undergo deformation easily with less effort. The following figures illustrate what are the typical characteristics involved in the material.

For easy deformation of metal the stress strain curve should have:

1. Lower yield point.
2. Gentle slope.
3. Larger elongation behavior.

1. Stress strain curve should have Lower yield point.

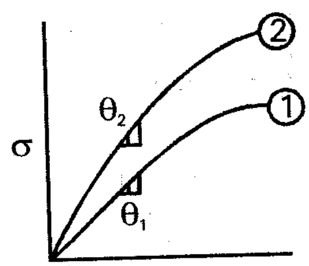
The load required for deformation is directly proportional to the yield point. Hence, if the yield point is high, higherload is required and lower the yield point of the material, lower is the loads required for deformation.The material with lower yield point can be easily shaped.
In the figure material 2 has the lower yield point as compared to 1. Hence, it is easier to deform material 2.
Whenever a material is heated to higher temperature the yield point is reduced and it becomes easier to deform.



σ - ∈ Curve for easy deformation

2. Stress strain should have Gentle slope.

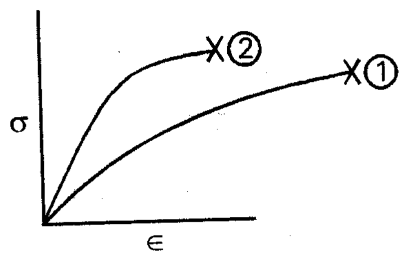
The stress strain curve should have lower gradient ie., gentle slope. It means the stiffness of the material must be low.
Stress strain curve with lower gradient will have gentle slope. Gentle slope needs lesser strain rate and hence lower rate of loading.
In figure material 1 has lower slope as compared to 2. Hence, material 1 is easier to deform.



Slope of σ - ∈ Curve

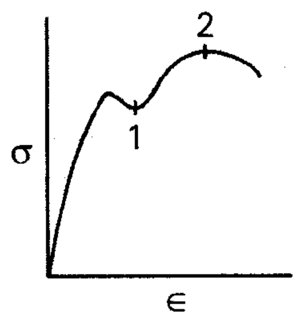
3. Stress Strain curve should have Larger elongation behavior.

A material with larger elongation will undergo more deformation without undergoing fracture, and it is extremely to shape the material.
In the figure material 1has larger elongation as compared to2. Hence, material1 can be easily deformed.



σ-∈ Curve with larger elongation

Strain hardening Type :
Some materials undergo strain hardening which means higher loads are required for deformation and more resistance is offered by the material. In the stress strain curve the strain hardening portion is represented by 12. If the slope12 is high, strain hardening of the material is more and it becomes difficult to deform.
By heating the material it can be softened and strain hardening is eliminated.



σ-∈ showing strain hardening

Unit1 (Class5) Introduction and Concepts

What we learnt in the last class

We learnt about advantages and limitations of MW process, concept of cold working, cold working vs properties, different stress-strain curves, nature of stress strain curve for easy deformation, concept of strain hardening.

What we are going to learn in today's class

concept of stress, strain, deformation, conventional stress and strain, true stress and strain, relationship between true stress and conventional stress, true strain and conventional strain, concept of flow stress, flow stress equation, generalized Hooke's Law, uniaxial, biaxial, and triaxial stresses their representation, stress tensor.

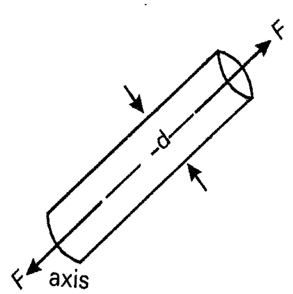
What is stress?

Stress is defined as the load acting on a material per unit area of the cross section.

It is the intensity of the load per unit area. When the load is acting normal to the area of cross section then the stress acting on the area is referred to as "Normal Stress". As the load on the cross section is increased, the value of stress also increases.

Then stress

$$\sigma = \frac{F}{Area} = \frac{F}{\left(\pi/4\right) d^2} = \frac{4F}{d^2}$$



A Cylindrical bar with axial force

What is Strain?

Strain is defined as the ratio of change in dimension to its original dimension. It signifies how much change in dimensions h as occurred under the influence of external load. The strain in the material under the influence of a normal load is referred to as "Normal Strain".

As the load on the material is increased the value of strain also increases.

What is Deformation?

When an external load is applied on the material, it will undergo changes in the dimensions and change in shape will take place. As a result strain will be induced in the material. The change in dimensions or the shape is referred to as "deformation".

Concept of Stress

As defined earlier stress is defined as the force per unit area of the body. The stress is assumed to be distributed uniformly over the cross section.

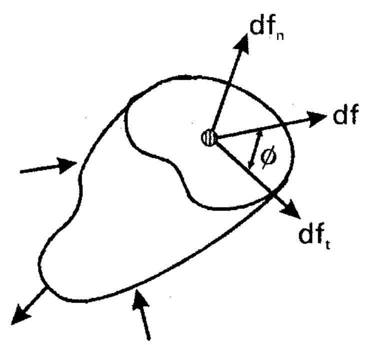
Consider a circular bar of uniform cross section

Of diameter 'd' is subjected to tensile force 'F' acting along the axis of the bar, then the average stress "σ" induced across any transverse cross section(C.S) perpendicular to the axis of the bar as shown:

In general a structural member or an element will not possess uniform geometry and the loads acting will be complex. In such cases one has to introduce the concept of state of stress at a point.

Consider a point 'O' within the body. Let a section plane pass through it and cut the body.
Let 'df' be the force acting on an area 'dA' of the cut section.
Now the stress acting at a point is defined as

$$\underset{dA \to o}{Lt} \frac{df}{dA} = \sigma$$



Resolution of a force

Concept of convention stress and conventional strain, true stress and true strain and their relationship.

Conventional stress: It is defined as the ratio of external load to the original area of cross section. It is also referred to as Engineering stress.

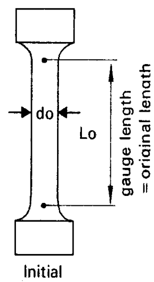
Conventional strain: It is defined as the ratio of change in length to its original length. However, the area of cross section and the length keeps on changing continuously under the influence of the external load. Hence, new terms true stress and true strain are defined as follows.

True stress: It is defined as the ratio of the external load to the instantaneous area of cross section.

True strain: It is defined as the ratio of change in length to the instantaneous previous length. For many of the engineering applications conventional stress and conventional strain data is sufficient. For generating true stress and true strain very sophisticated instruments are required, takes lot of time and very tedious.
Consider a bar subjected to tensile test. Let the initial diameter be d_0 and initial length be L_0. Let it be loaded gradually from P_1 , P_2 ,P_3 ………P_n. Now the bar will undergo changes in the diameter and length. The diameter will reduce and the length will increase gradually. Let the change in diameters be d_1,d_2,d_3,….d_n and the corresponding change in lengths be

L_1 ,L_2 ,L_3 ……L_n . and the corresponding areas be

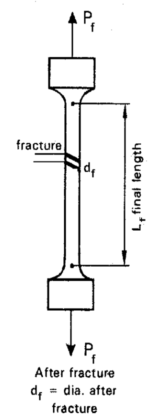
A_1, A_2 ,A_3 …….A_n . Now one can calculate the conventional stress, conventional strain, true stress, true strain and their relationships.



Specimen before testing

$$A_0 = \frac{\pi}{4} d_0^2$$

$$A_f = \frac{\pi}{4} d_f^2$$



Specimen after testing

Conventional Stress

$$\sigma_{C_1} = \frac{P_1}{A_0}$$

$$\sigma_{C_2} = \frac{P_2}{A_0}$$

$$\sigma_{C_3} = \frac{P_3}{A_0}$$

Conventional Strain

$$\epsilon_{C_1} = \frac{L_1 - L_0}{L_0}$$

$$\epsilon_{C_2} = \frac{L_2 - L_0}{L_0}$$

$$\epsilon_{C_3} = \frac{L_3 - L_0}{L_0}$$

True Stress

$$\sigma_{t_1} = \frac{P_1}{A_1}$$

$$\sigma_{t_2} = \frac{P_2}{A_2}$$

$$\sigma_{t_3} = \frac{P_3}{A_3}$$

.
.
.
.

True Strain

$$\epsilon_{t_1} = \frac{L_1 - L_0}{L_0}$$

$$\epsilon_{t_2} = \frac{L_2 - L_1}{L_1}$$

$$\epsilon_{t_3} = \frac{L_3 - L_2}{L_2}$$

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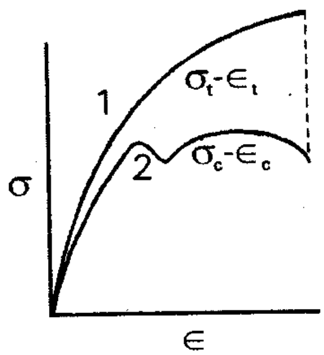


$\sigma_t - \epsilon_t$ ***and***
$\sigma_c - \epsilon_c$ ***curves***

$$\text{Conv Stress } \sigma_c = \frac{Load}{\text{original area of CS}} = \frac{P}{A_0}$$

$$\text{Conventional Strain } \epsilon_c = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{Lo} = \frac{L_f - L_0}{L_0}$$

$$\text{or } \frac{L_f}{L_o} = (1 + \epsilon_c)$$

Volume of the bar is constant

$$V = A_0 L_0 = A_1 L_1 = A_2 L_2 \ldots\ldots\ldots A_f L_f$$

$$\frac{A_o}{A_f} = \frac{L_f}{L_o}$$

From the above equations

$$\frac{A_o}{A_f} = 1 + \epsilon_C$$

$$\text{True Stress } \sigma_t = \frac{Load}{\text{Instantaneous area of C.S}} = \frac{P}{A_i}$$

$$\text{Can be written as } \sigma_t = \frac{P}{A_0} \times \frac{A_0}{A_i}$$

$$\text{but } P/A_o = \sigma_c$$

$$\therefore \sigma_t = \sigma_c (1 + \epsilon_c)$$

$$\text{True strain } \epsilon_t = \frac{\text{Change in length}}{\text{Instantaneous length}} = \int_{L_o}^{L_f} \frac{\Delta L}{L_i}$$

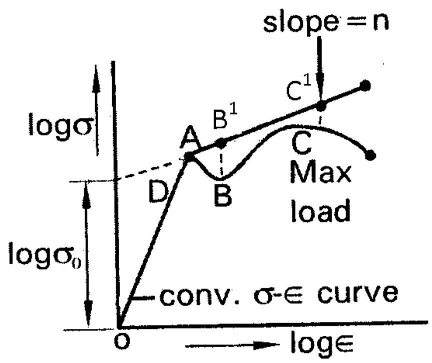
$$= \ln\left(\frac{L_f}{L_o}\right)$$

$$\therefore \epsilon_t = \ln\left(\frac{A_o}{A_f}\right) \text{ or } \ln(1 + \epsilon_c)$$

$$\text{ie., } \epsilon_t = \ln(1 + \epsilon_c)$$

Concept of Flow stress

The stress required for plastic deformation or to make the metal flow to any given strain can be obtained from the true stress true strain curve. True stress true strain curve gives an idea about how the metal can flow plastically. Hence, it is referred to as "Flow Curve".



Flow Curve

Consider the stress strain curve for a metal from above the yield point ie., from the start of plastic flow to the maximum load at which the metal begins to form neck (BC). BC represent strain hardening region.

Taking log values of σ and ε for portion B to C, a stress strain curve can be plotted, which is a straight line B^1C^1.

If this straight line B^1C^1 is extended back, it will intercept the σ axis at D.

Let 'n' be the slope of the line B^1C^1 which represents strain hardening portion BC of the stress strain curve. This slope n, is referred to as "strain hardening coefficient".

Let the straight line B^1C^1 cut the σ axis at the origin.

Let Log σ_0 be the intercept as shown in the figure.

Then $\text{Log } \sigma = \text{Log } \sigma_0 + n \text{ Log } \varepsilon$ is the equation for a straight line.

ie., equation of the Stress at any point on the line.

$$\text{Log } \sigma = \text{Log } \sigma_0 + \text{Log } \varepsilon^n$$

$$\text{Log } \sigma = \text{Log } \sigma_0 \varepsilon^n$$

$$\text{or} \quad \sigma = \sigma_0 \varepsilon^n$$

This equation is referred to as "Flow Stress Equation".

Generalized Hooke's Law

Hooke's law in 1D states that Stress is proportional to strain within the proportionality limit.

$\sigma \propto \epsilon$ or $\sigma = E\epsilon$ or $\epsilon = \sigma/E$

where E is constant of proportionality called as Young's Modulus or Modulus of Elasticity.

Consider a bar subjected to uniaxial tensile load. The length of the bar increases and the diameter decreases proportionately. Due to this there will be longitudinal strain along the length and lateral strain across the diameter of the bar. There exists a relation between these two types of strains which is referred to as "Poissons Ratio". It is defined as the ratio of Lateral strain to the longitudinal strain.

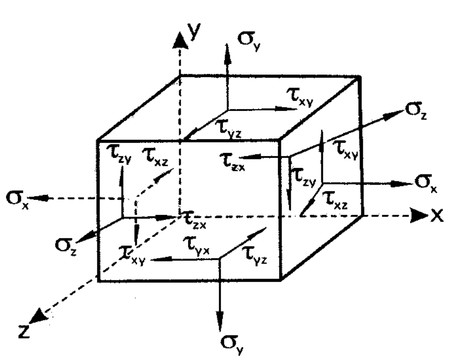
Poissons Ratio = (Lateral Strain/Longitudinal Strain)
(γ)

$\gamma = \epsilon_{lat}/L_{ong}$

In a 3D stress system there are 6 rectangular components of stresses σ_x σ_y σ_z

In a 3D stress system there are 6 rectangular components of stress (σ_x σ_y σ_z τ_{xy} τ_{yz} τ_{zx}) and 6 rectangular component of strain. (ϵ_x ϵ_y ϵ_z γ_{xy} γ_{yz} γ_{zx}) respectively. Taking complimentary shear stresses to be equal

$$\tau_{xy} = \tau_{yz}$$
$$\tau_{yz} = \tau_{zy}$$
$$\tau_{zx} = \tau_{xz}$$



3D Stress element

In matrix form it is written as

$$\sigma_{ij} \text{ or } \tau_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$