Lecture 1- 4: Closed Coiled helical Spring

Closed Coiled helical springs subjected to axial loads:

Definition: A spring may be defined as an elastic member whose primary function is to deflect or distort under the action of applied load; it recovers its original shape when load is released.

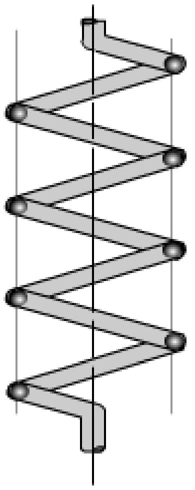
or

Springs are energy absorbing units whose function is to store energy and to restore it slowly or rapidly depending on the particular application.

Important types of springs are:

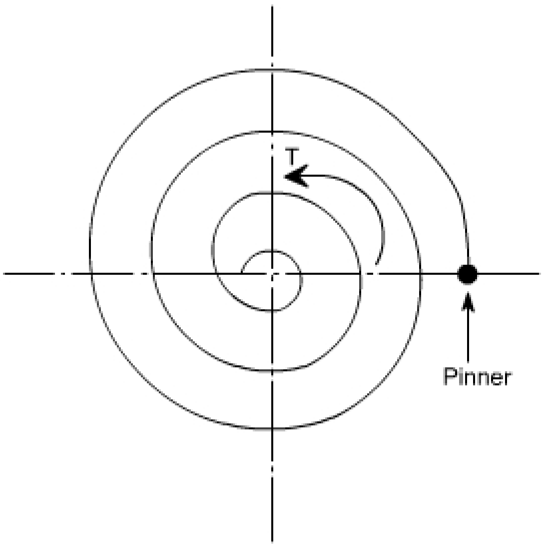
There are various types of springs such as

(i) helical spring: They are made of wire coiled into a helical form, the load being applied along the axis of the helix. In these type of springs the major stresses is torsional shear stress due to twisting. They are both used in tension and compression.

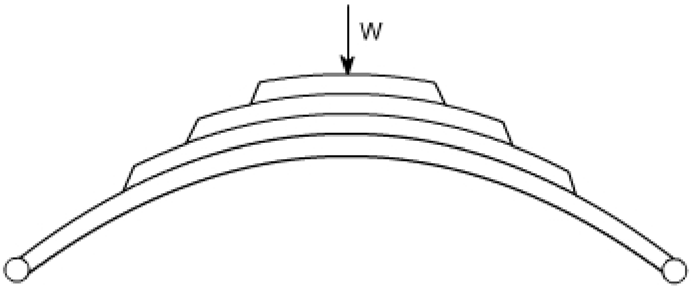


(ii) Spiral springs: They are made of flat strip of metal wound in the form of spiral and loaded in torsion.

In this the major stresses are tensile and compression due to bending.



(iv) Leaf springs: They are composed of flat bars of varying lengths clamped together so as to obtain greater efficiency . Leaf springs may be full elliptic, semi elliptic or cantilever types, In these type of springs the major stresses which come into picture are tensile & compressive.



These type of springs are used in the automobile suspension system.

Uses of springs :

(a) To apply forces and to control motions as in brakes and clutches.

(b) To measure forces as in spring balance.

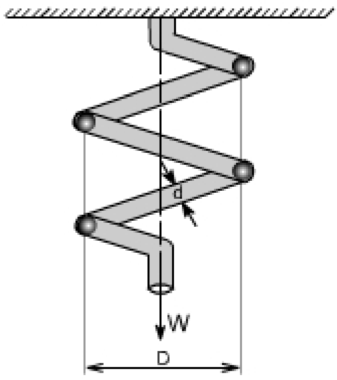
(c) To store energy as in clock springs.

(d) To reduce the effect of shock or impact loading as in carriage springs.

(e) To change the vibrating characteristics of a member as inflexible mounting of motors.

Derivation of the Formula :

In order to derive a necessary formula which governs the behaviour of springs, consider a closed coiled spring subjected to an axial load W.



Let

W = axial load

D = mean coil diameter

d = diameter of spring wire

n = number of active coils

C = spring index = D / d For circular wires

l = length of spring wire

G = modulus of rigidity

x = deflection of spring

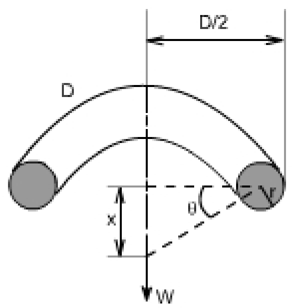
q = Angle of twist

when the spring is being subjected to an axial load to the wire of the spring gets be twisted like a shaft.

If q is the total angle of twist along the wire and x is the deflection of spring under the action of load W along the axis of the coil, so that

x = D / 2 .

again l = D n [consider ,one half turn of a close coiled helical spring]



Assumptions: (1) The Bending & shear effects may be neglected

(2) For the purpose of derivation of formula, the helix angle is considered to be so small that it may be neglected.

Any one coil of a such a spring will be assumed to lie in a plane which is nearly □r to the axis of the spring. This requires that adjoining coils be close together. With this limitation, a section taken perpendicular to the axis the spring rod becomes nearly vertical. Hence to maintain equilibrium of a segment of the spring, only a shearing force V = F and Torque T = F. r are required at any X – section. In the analysis of springs it is customary to assume that the shearing stresses caused by the direct shear force is uniformly distributed and is negligible
so applying the torsion formula.
Using the torsion formula i.e

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$$

$$\text{and substitituting } J = \frac{\pi d^4}{32}; T = w.\frac{d}{2}$$

$$\theta = \frac{2.x}{D}; l = \pi D.x$$

<u>SPRING DEFLECTION</u>

$$\frac{w.d/2}{\frac{\pi d^4}{32}} = \frac{G.2x/D}{\pi D.n}$$

Thus,

$$x = \frac{8w.D^3.n}{G.d^4}$$

Spring striffness: The stiffness is defined as the load per unit deflection therefore

$$k = \frac{w}{x} = \frac{w}{\frac{8w.D^3.n}{G.d^4}}$$

Therefore

$$k = \frac{G.d^4}{8.D^3.n}$$

<u>Shear stress</u>

$$\frac{w.d/2}{\frac{\pi d^4}{32}} = \frac{\tau_{max^m}}{d/2}$$

$$\text{or } \tau_{max^m} = \frac{8wD}{\pi d^3}$$

WAHL'S FACTOR :

In order to take into account the effect of direct shear and change in coil curvature a stress factor is defined, which is known as Wahl's factor

K = Wahl' s factor and is defined as $K = \frac{4c-1}{4c-4} + \frac{0.615}{c}$

Where C = spring index

= D/d

if we take into account the Wahl's factor than the formula for the shear stress becomes $\tau_{max^m} = \frac{16.T.k}{\pi d^3}$

Strain Energy : The strain energy is defined as the energy which is stored within a material when the work has been done on the material.

In the case of a spring the strain energy would be due to bending and the strain energy due to bending is given by the expansion

$$U = \frac{T^2 L}{2EI}$$

$$L = \pi D n$$

$$I = \frac{\pi d^4}{64}$$

so after substitution we get

$$U = \frac{32T^2 Dn}{E.d^4}$$

Example: A close coiled helical spring is to carry a load of 5000N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm^2 .if the number of active turns or active coils is 8.Estimate the following:

(i) wire diameter

(ii) mean coil diameter

(iii) weight of the spring.

Assume G = 83,000 N/mm^2 ; □ = 7700 kg/m^3

solution :

(i) for wire diametre if W is the axial load, then

$$\frac{\frac{w.d/2}{\pi d^4}}{32} = \frac{\tau_{max^m}}{d/2}$$

$$D = \frac{400}{d/2} \cdot \frac{\pi d^4}{32} \cdot \frac{2}{W}$$

$$D = \frac{400.\pi d^3.2}{5000.16}$$

$$D = 0.0314\, d^3$$

Futher, deflection is given as

$$x = \frac{8wD^3.n}{G.d^4}$$

on substituting the relevant parameters we get

$$50 = \frac{8.5000.(0.0314\,d^3)^3.8}{83{,}000.d^4}$$

$d = 13.32mm$

Therefore,

D = .0314 x $(13.317)^3$mm

=74.15mm

D = 74.15 mm

Weight

mass or weight = volume. density

= area.length of the spring.density of spring material

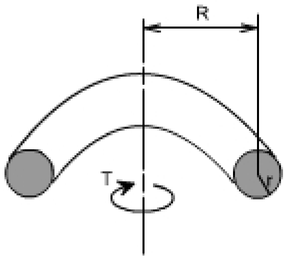
$$= \frac{\pi d^2}{4}.\pi Dn.\rho$$

On substituting the relevant parameters we get

Weight = 1.996 kg

= 2.0kg

Close – coiled helical spring subjected to axial torque T or axial couple.



In this case the material of the spring is subjected to pure bending which tends to reduce Radius R of the coils. In this case the bending moment is constant through out the spring and is equal to the applied axial Torque T. The stresses i.e. maximum bending stress may thus be determined from the bending theory.

$$\sigma_{max} = \frac{M.y}{I} = \frac{T.d/2}{\frac{\pi d^4}{64}}$$

$$\sigma_{max} = \frac{32T}{\pi d^3}$$

Deflection or wind – up angle:

Under the action of an axial torque the deflection of the spring becomes the “wind – up” angle of the spring which is the angle through which one end turns relative to the other. This will be equal to the total change of slope along the wire, according to area – moment theorem

$$\theta = \int_0^L \frac{MdL}{EI} \quad \text{but } M = T$$

$$= \int_0^L \frac{T.dL}{EI} = \frac{T}{EI}\int_0^L dL$$

Thus, as 'T' remains constant

$$\theta = \frac{T.L}{EI}$$

Futher

$$L = \pi D.n$$

$$I = \frac{\pi d^4}{64}$$

Therefore, on substitution, the value of θ obtained is

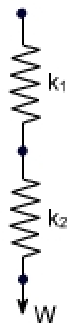
$$\boxed{\theta = \frac{64T.D.n}{E.d^4}}$$

Springs in Series: If two springs of different stiffness are joined endon and carry a common load W, they are said to be connected in series and the combined stiffness and deflection are given by the following equation.

$$\frac{W}{k} = x_1 + x_2 = \frac{W}{k_1} + \frac{W}{k_2}$$

or

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$



Springs in parallel: If the two spring are joined in such a way that they have a common deflection 'x' ; then they are said to be connected in parallel.In this care the load carried is shared between the two springs and total load $W = W_1 + W_2$

$$x = \frac{W}{k} = \frac{W_1}{k_1} = \frac{W_2}{k_2}$$

$$\text{Thus } W_1 = \frac{Wk_1}{k}$$

$$W_2 = \frac{Wk_2}{k}$$

Futher

$$W = W_1 + W_2$$

$$\text{thus } \boxed{k = k_1 + k_2}$$