STEAM AND GAS TURBINES

1.0 Introduction:

Steam turbines use the steam as a working fluid. In steam turbines, high pressure steam from the boiler is expanded in nozzle, in which the enthalpy of steam being converted into kinetic energy. Thus, the steam at high velocity at the exit of nozzle impinges over the moving blades (rotor) which cause to change the flow direction of steam and thus cause a tangential force on the rotor blades. Due to this dynamic action between the rotor and the steam, thus the work is developed.

These machines may be of axial or radial flow type devices.

Steam turbines may be of two kinds, namely, (i) impulse turbineand (ii)Reaction turbine.

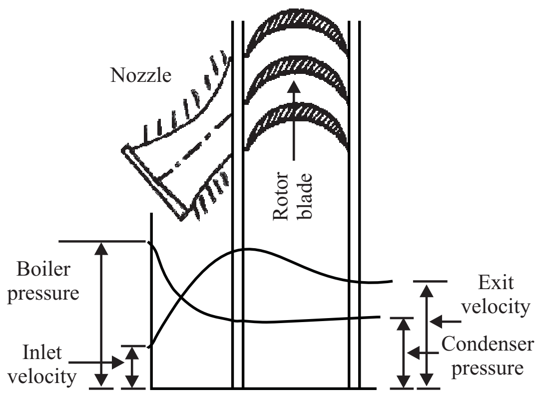


Fig.1 Impulse turbine

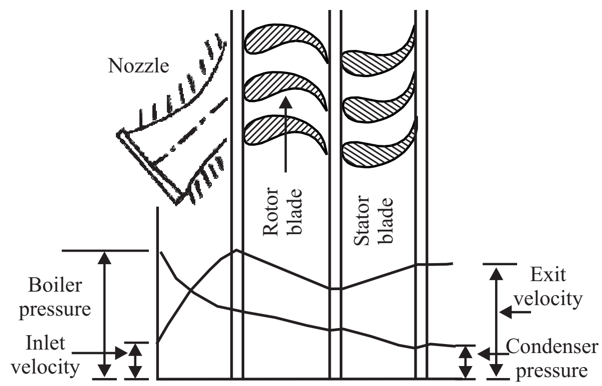


Fig.2 Reaction turbine

In Impulse turbine, the whole enthalpy drop (pressure drop) occurs in the nozzle itself. Hence pressure remain constant when the fluid pass over the rotor blades. Fig.1 shows the schematic diagram of Impulse turbine.

In Reaction turbines, in addition to the pressure drop in the nozzle there will also be pressure drop occur when the fluid passes over the rotor blades. Fig.2 shows the Reaction turbine .

Most of the steam turbine are of axial flow type devices except Ljungstrom turbine which is a radial type.

2.0 Difference between Impulse & Reaction Steam Turbine :

Sl. no.	Particulars	Impulse turbine	Reaction Turbine
1	Steam Expansion	Expands fully in nozzle and pressure remain constant over the rotor blades.	Expands partially in nozzle and further expansion takes place when it passes over the rotor blades.

2	Relative velocity	Remains constant i.e., $V_{r2}=V_{r1}$	Go on increasing as the pressure drop occurs on the moving blades, i.e., $V_{r2} > V_{r1}$
3	Blade sections	Blades are symmetrical	Blades are of aero-foil type.
4	Steam velocity at the inlet of machine	Very high	Moderate or low.
5	Blade efficiency or Stage efficiency	Comparatively low.	High.
6	Number of stages required for given pressure drop or power output	Few stages	More number of stages
7	Suitability	Suitable, where the efficiency is not a matter of fact.	Suitable, where the efficiency is a matter of fact.

3.0 Compounding of Steam Turbines

If only one stage, comprising of single nozzle and single row of moving blades, is used, then the flow energy or kinetic energy available at inlet of machine is not absorbed fully by one row of moving blades running at half of absolute velocity of steam entering the stage (because, even for maximum utilization, $U/V = \phi = 1/2$). It means to say that, due to high rotational speed all the available energy at inlet of machine is not utilized which being simply wasted at exit of machine. Also we know that for maximum utilization, exit velocity of steam should be minimum or negligible. Hence, for better utilization, one has to have a reasonable tangential speed of rotor. In order to achieve this, we have to use, two or more rows of moving blades with a row of stationary blades between every pair of them. Now, the total energy of steam available at inlet of machine can be absorbed by all the rows in succession until the kinetic energy of steam at the end of the last row becomes negligible.

Hence **compounding** can be defined *as the method of obtaining reasonable tangential speed of rotor for a given overall pressure drop by using more than one stages.* Compounding is necessary for steam turbines because if the tangential blade tip velocity greater than 400 m/s, then the blade tips are subjected to centrifugal stress. Due this, utilization is low hence the efficiency of the stage is also low.

Compounding can be done by the following methods, namely, (i)Velocity compounding, (ii) Pressure compounding or Rateau stage (iii) Pressure-Velocity compounding and (iv) Impulse- Reaction staging.

3.1 Velocity Compounding (Curtis Stage) of Impulse Turbine :

This consists of set of nozzles, rows of moving blades (rotor) & a rows of stationary blades (stator). Fig.3 shows the corresponding velocity compounding Impulse Turbine. The function of stationary blades is to direct the steam coming from the first moving row to the next moving row without appreciable change in velocity. All the kinetic energy available at the nozzle exit is successively absorbed by all the moving rows & the steam is sent from the last moving row with low velocity to achieve high utilization. The turbine works under this type of compounding stage is called velocity compounded turbine.E.g. **Curtis stage steam turbine**.

3.2 Pressure Compounded (Rateau Stage) Impulse Turbine :

A number of simple impulse stages arranged in series is called as pressure compounding. In this case, the turbine is provided with rows of fixed blades which acts as a nozzles at the entry of each rows of moving blades. The total pressure drop of steam does not take place in a single nozzle but divided among all the rows of fixed blades which act as nozzle for the next moving rows. Fig.4 shows the corresponding pressure compounding Impulse turbine .

Pressure compounding leads to higher efficiencies because very high flow velocities are avoided through the use of purely convergent nozzles. For maximum utilization, the absolute velocity of steam at the outlet of the last rotor must be axially directed. It is usual in large turbines to have pressure compounded or reaction stages after the velocity compounded stage.

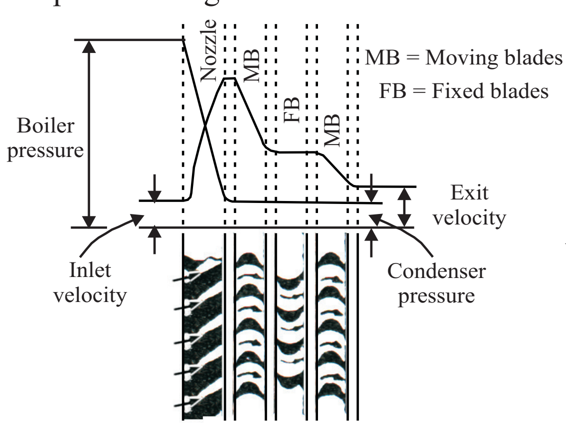


Fig.3 Velocity compounded impulse turbine

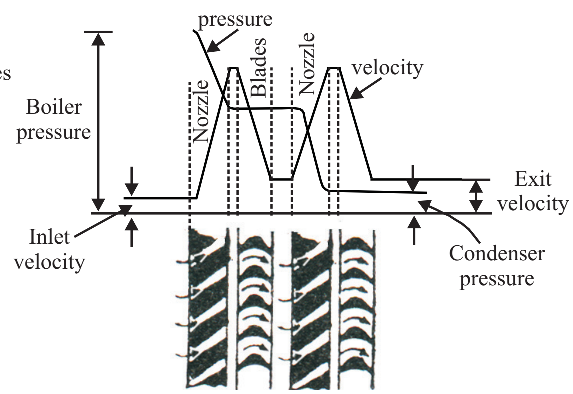


Fig.4 Pressure compounded impulse turbine

3.3 Pressure -Velocity Compounding

In this method, high rotor speeds are reduced without sacrificing the efficiency or the output. Pressure drop from the chest pressure to the condenser pressure occurs at two stages. This type of arrangement is very popular due to simple construction as compared to pressure compounding steam turbine.

Pressure-Velocity compounding arrangement for two stages is as shown in Fig.5. First and second stage taken separately are identical to a velocity compounding consists of a set of nozzles and rows of moving blades fixed to the shaft and rows of fixed blades to casing. The entire expansion takes place in the nozzles. The high velocity steam parts with only portion of the kinetic energy in the first set of the moving blades and then passed on to fixed blades where only change in direction of jet takes place without appreciable loss in velocity. This jet then passes on to another set of moving vanes where further drop in kinetic energy occurs. This type of turbine is also called Curtis Turbine.

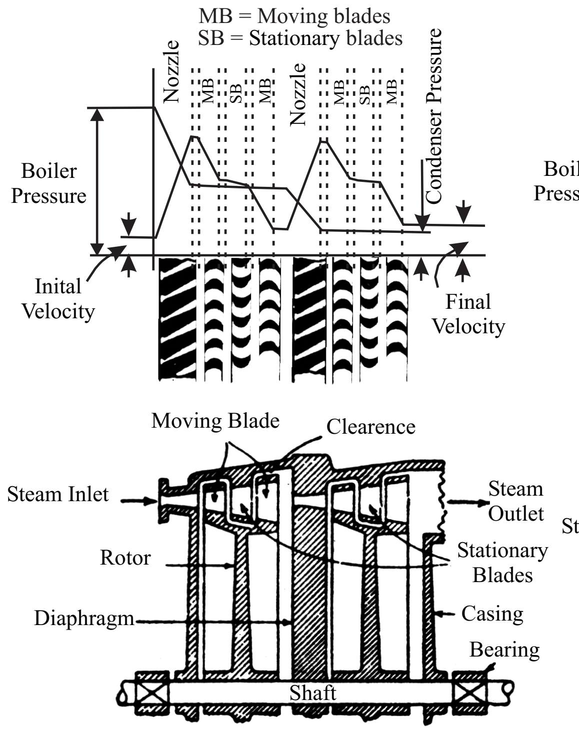


Fig.5 Pressure - Velocity compounding

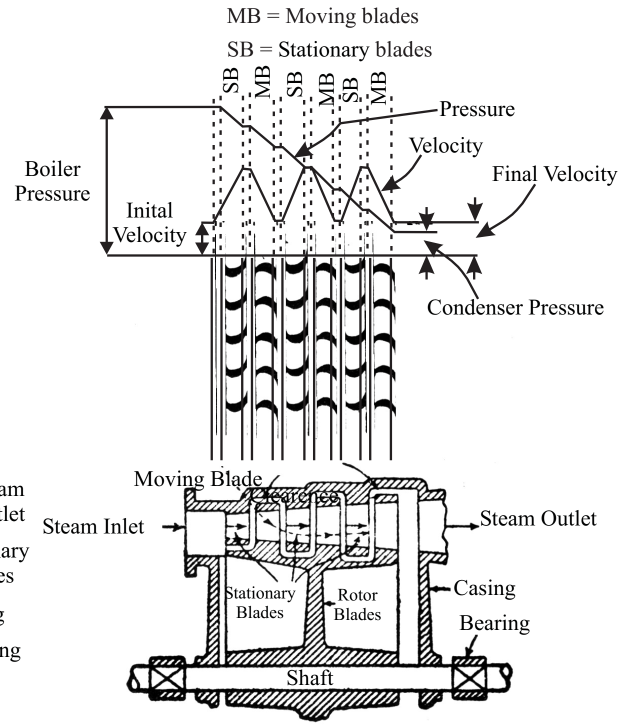


Fig.6 Impulse Reaction turbine

3.4 Impulse- Reaction Turbine

In this type of turbine there is application of both principles namely impulse and reaction. This type of turbine is shown in Fig.6. The fixed blades in this arrangement corresponding to the nozzles referred in the impulse turbine. Instead of a set of nozzles, steam is admitted for whole of the circumference. In passing through the first row of fixed blades, the steam undergoes a small drop in pressure and its velocity increases. Steam then enters the first row of moving blades as the case in impulse turbine it suffers a change in direction and therefore momentum. This gives an impulse to the blades. The pressure drop during this gives rise to reaction in the direction opposite to that of added velocity. Thus the driving force is vector summation of impulse and reaction.

Normally this turbine is known as Reaction turbine. The steam velocity in this type of turbine is comparatively low, the maximum being about equal to blade velocity. This type of turbine is very successful in practice. It is also called as Parson's Reaction turbine.

4.0 Effects of Blade and Nozzle Losses:

The losses in flow over blades due to friction, leakage and turbulence are not negligible in most of the cases. These losses reduce the velocity obtained at the outlet of blade and is less than the velocity which would be obtained in loss-free flow.

The ratio of the actual velocity at the exit of the flow passage to the ideal exit velocity is called *blade velocity coefficient or nozzle velocity coefficient.* Thus, $C_b = V_e / V^1_e$

Where V_e and V^1_e are the actual and ideal velocities of the nozzle or stator or rotor blades.

For Impulse turbine, $V_{r2} = V_{r1}$ in ideal conditions. If the losses are considered, then,

$$C_b = V_{r2}/V_{r1}$$

5.0 Analysis on Single Stage Impulse Turbine :

5.1 General Velocity Diagrams for Impulse Turbine :

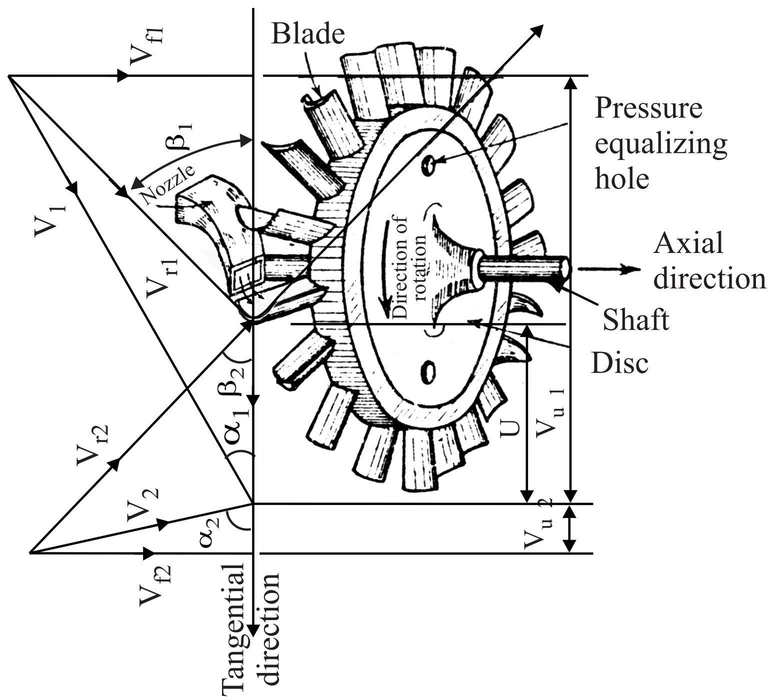


Fig. 7 Steam flow through blade passages & velocity diagrams.

A schematic diagram of an Impulse wheel along with combined velocity diagram is shown in Fig.7.

5.2 Forces on the Blade and Work Done :

There are two types of forces occurring on the rotor blades and shaft respectively. They are:

(a) Tangential force due to change in whirl velocity or tangential component of absolute velocities

$$F_T = \frac{\dot{m}}{g_c}\left(V_{u1} \pm V_{u2}\right) \text{ Newtons} \qquad (1)$$

Here positive sign is used, as usual, if V_{u1} and V_{u2} are opposite to each other and negative sign if they are in the same direction.

(b) Axial thrust due to change in axial velocity components is

$$F_a = \frac{\dot{m}}{g_c}(V_{f1} - V_{f2}) \text{ Newtons} \quad (2)$$

5.3 Blade Efficiency or Diagram Efficiency (η_b):

It is defined *as the ratio of work done per kg of steam by the rotor (output by the rotor) to the energy available at the inlet per kg of steam.* Thus,

$$\eta_b \text{ or } \eta_r = \frac{\text{Work done / kg of steam by the rotor}}{\text{Energy available / kg of steam at Inlet.}}$$

$$\eta_b = \frac{W}{V_1^2/2g_c} = \frac{2U(V_{u1} \pm V_{u2})}{V_1^2} = \frac{2U\Delta V_u}{V_1^2} \quad (3)$$

5.4 The Stage Efficiency (η_s):

It is defined as "*the ratio of work done per kg of steam by the rotor to the isentropic enthalpy change per kg of steam in the nozzle*".

$$\therefore \ \eta_{stage} = \frac{W}{\Delta h_0^1} = \frac{U\Delta V_u/g_c}{\Delta h_0^1}$$

Or
$$\eta_{stage} = \frac{(U\Delta V_u)/g_c}{V_1^2/2g_c} \times \frac{(V_1^2)/2g_c}{\Delta h_0^1}$$

$$\eta_{stage} = \eta_{blade} \times \eta_{nozzle} \quad (4)$$

5.5 Condition for Maximum Utilization Factor or Blade efficiency with Equiangular Blades for Impulse Turbine :

Condition for maximum utilization factor or blade efficiency with equiangular blades for Impulse turbine and the influence of blade efficiency on the steam speed in a single stage Impulse turbine can be obtained by considering corresponding velocity diagrams as shown in Fig.9. Due to the effect of blade friction loss, the relative velocity at outlet is reduced than the relative velocity at inlet. Therefore, $V_{r2} = C_b V_{r1}$, corresponding to this condition, velocity triangles (qualitative only) are drawn as shown in Fig.9.

Energy transfer as a work per kg of steam is,

$$E = W = \frac{U(V_{u1}+V_{u2})}{g_c} = \frac{U\Delta V_u}{g_c}$$

From velocity triangle :

$$\Delta V_u = V_{u1} + V_{u2} = V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2$$

$$= V_{r1}\cos\beta_1\left(1 + \frac{V_{r2}}{V_{r1}}\frac{\cos\beta_2}{\cos\beta_1}\right)$$

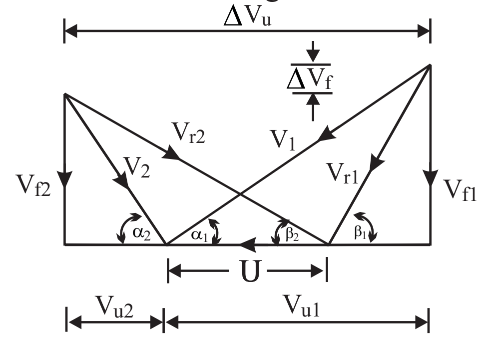


Fig.9. Effect of blade friction

But $V_{r1}\cos\beta_1 = V_1\cos\alpha_1 - U$

$\therefore\ \Delta V_u = (V_1 \cos\alpha_1 - U)[1 + (V_{r2}/V_{r1})(\cos\beta_2/\cos\beta_1)]$

But $\frac{V_{r2}}{V_{r1}} = C_b$ and for a given rotor, β_1 & β_2 are also fixed. Let $K = \frac{\cos\beta_2}{\cos\beta_1}$

$\therefore\ \Delta V_u = (V_1 \cos\alpha_1 - U)[1 + C_b K]$

$\therefore$ Work done per kg of steam is,

$$W = U(V_1 \cos\alpha_1 - U)[1 + C_b K] \tag{5}$$

If we express in terms of speed ratio, $\phi = U/V_1$, we get

$$W = V_1^2\ \phi\ [\cos\alpha_1 - \phi](1 + C_b K) \tag{6}$$

The available energy per kg of steam at inlet is $V_1^2/2g_c$

Also, blade efficiency, $\eta_b = \frac{W}{V_1^2/2g_c} = \frac{2\phi V_1^2[\cos\alpha_1 - \phi](1 + C_b K)}{V_1^2}$

$$\eta_b = 2(\phi \operatorname{Cos}\alpha_1 - \phi^2](1 + C_b K) \tag{7}$$

Eqn.(7) shows that the rotor efficiency or blade efficiency varies parabolically for the given α_1, C_b and K.

For the maximum blade efficiency, $\frac{d\eta_b}{d\phi} = 0$

$$\therefore \text{ Speed ratio, } \phi = \frac{\cos\alpha_1}{2} \tag{8}$$

It is same as the maximum utilization for an impulse turbine discused in chapter-2. The maximum blade efficiency becomes

$$\therefore\ \eta_{b,max} = 2\left(\frac{\cos^2\alpha_1}{2} - \frac{\cos^2\alpha_1}{4}\right)(1 + C_b K)$$

$$\eta_{b,\ max} = \frac{\cos^2\alpha_1}{2}\ (1 + C_b K) \tag{9}$$

Further, if rotor angles are equiangular i.e., $\beta_1 = \beta_2$ and $V_{r1} = V_{r2}$

$$\text{Then, } \eta_{b,max} = \cos^2\alpha_1 \tag{10}$$

It is seen that the $\eta_{b,\ max}$ is same the as maximum utilization for Impulse turbine when there is no loss and blades are equiangular.

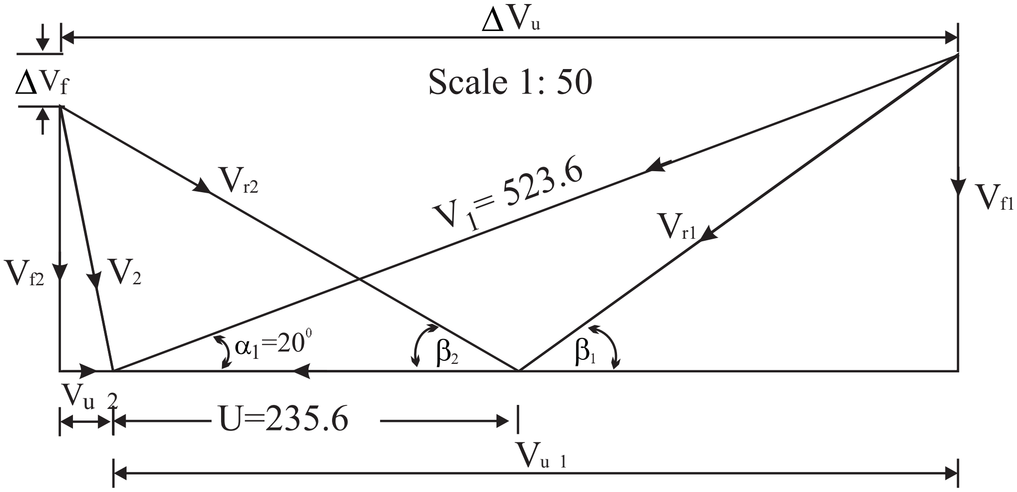
Example 1 : A single stage Impulse turbine has a diameter of 1.5 m. and running at 3000 RPM. The nozzle angle is 20^{O}. Speed ratio is 0.45. Ratio of relative velocity at the outlet to that at inlet is 0.9. The outlet angle of the blade is 3^{O} less than inlet angle. Steam flow rate is 6 kg/s. Draw the velocity diagrams and find the following. (i) Velocity of whirl (ii) Axial thrust (iii) Blade angles (iv) Power developed.

Solution : Given : D=1.5 m, $\dot{m}$ = 6 kg/s, N=3000RPM, $\alpha_1 = 20^{O}$, $\phi = U/V_1 = 0.45$,

$C_b = V_{r2}/V_{r1} = 0.9$, $\beta_2 = \beta_1 - 3^{O}$, To Find : ΔV_u, F_a, β_1, β_2, P.

Tangential speed of rotor, $U = \dfrac{\pi DN}{60} = \dfrac{\pi \times 1.5 \times 3000}{60} = 235.6$ m/s

Velocity of steam at inlet, $V_1 = U/\phi = 235.6/0.45 = 523.6$ m/s



From Graph : $\beta_1 = 35^{O}$, V_{r1} = 312.5 m/s.

$V_{r2} = 0.9 \times 312.5 = 281.3$ m/s.

$\therefore \beta_2 = 35^{O} - 3^{O} = 32^{O}$

(i) Velocity of Whirl :

$\Delta V_u = 9.9$ cm x50 = 495 m/s

(ii) Axial thrust : $F_a = \dfrac{\dot{m}}{g_c} \times \Delta V_f$

$\Delta V_f = 0.6$ cm x 50 = 30 m/s

$F_a = 6 \times 30 = 180$ N.

(iii) Blade angles : $\beta_1 = 35^{O}$, $\beta_2 = 32^{O}$

(iv) Power developed: $P = \dfrac{\dot{m}}{g_c} U \Delta V_u$

$= (6 \times 235.6 \times 495)/1000$

P = 700 kW

(v) Stage efficiency : $\eta_s = \dfrac{2U \Delta V_u}{V_1^2}$

$= \dfrac{2 \times 235.6 \times 495 \times 100}{(523.6)^2}$

η_s = 85.1%

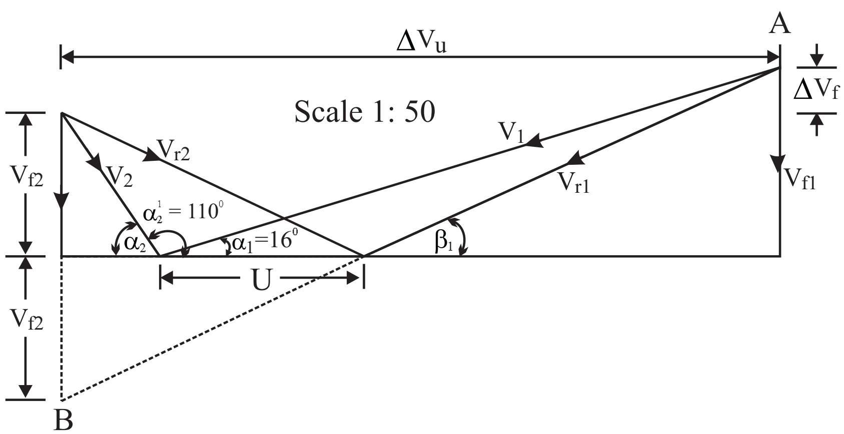
f**Example 2:** Steam flows through the nozzle with a velocity of 450 m/s at a direction which is inclined at an angle of 16^O to the wheel tangent. Steam comes out of the moving blades with a velocity of 100 m/s in the direction of 110^O with the direction of blade motion. The blades are equiangular and the steam flow rate is10 kg/s. Find:(i) Power developed, (ii)The power loss due to friction (iii)Axial thrust (iv) Blade efficiency and (v) Blade coefficient.

Solution :Given: $V_1 = 450$ m/s, $\alpha_1 = 16^o$, $V_2 = 100$ m/s, $\alpha_2^1 = 110^o$, $\alpha_2 = 70^O$ $\beta_1 = \beta_2$,

$\dot{m} = 10$ kg/s. *Find* : P, P_f, F_a, η_b, C_b.

Graph construction : For the selected scale, draw V_1 w.r.t α_1 and V_2 w.r.t α_2 and find the value of V_{f1} & V_{f2}. To get β_1 and β_2, produce V_{f2} in backward direction & draw the line from the apex 'A' of inlet velocity triangle which cuts at V_{f2} produced backward at B.

Now measure $\beta_1$and β_2 and find out the tangential velocity of rotor.



From Graph : $U = 167$ m/s, $V_{r1} = 293$ m/s, $V_{r2} = 222$ m/s

$$\beta_1 = \beta_2 = 25^O, \Delta V_f = 30.1 \text{ m/s}, \ \Delta V_u = 466.8 \text{ m/s}.$$

(i) Power developed, $P = \dfrac{\dot{m}}{g_c} \times U \Delta V_u = \dfrac{10 \times 167 \times 466.8}{1000} = 780$ kW

(ii) Power loss to friction : $P_f = \dfrac{\dot{m}}{g_c} \times \Delta h_f$

Δh_f = Pressure Energy loss due to friction in the rotor $= 1/2g_c [(V_{r1}^2 - V_{r2}^2)]$

$$\Delta h_f = \frac{(293^2 - 222.0^2)}{2 \times 1000} = 18.28 \text{ kJ/kg}$$

$\therefore$ Power loss, $P_f = \dot{m}\ \Delta h_f = 10 \times 18.28 = 182.8$ kW

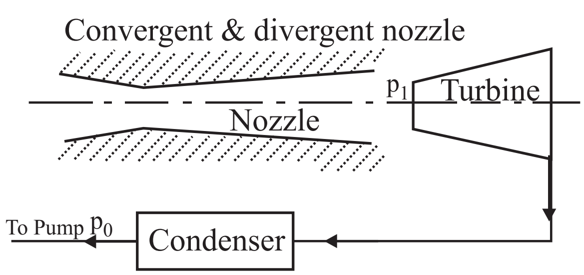
(iii) Axial thrust, $F_a = \dfrac{\dot{m}}{g_c} \Delta V_f = \dfrac{10 \times 30.1}{1} = 301$ N

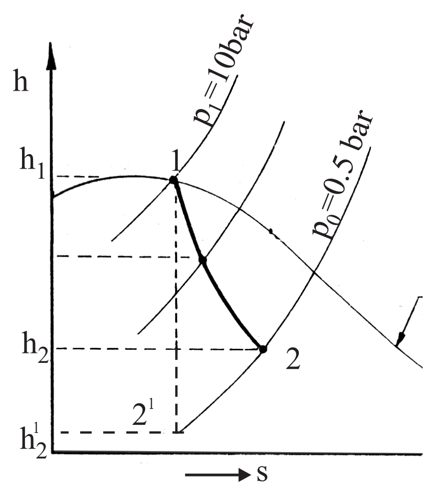
(iv) Rotor Efficiency : $\eta_b = \dfrac{2U\ \Delta V_u}{V_1^{\,2}} = \dfrac{2 \times 167 \times 466.8}{450^2} = 77\,\%$

(v) Blade velocity coefficient : $C_b = \dfrac{V_{r2}}{V_{r1}} = \dfrac{222}{293} = 0.758$

Example 3 : Dry saturated steam at 10 atmospheric pressure is supplied to single rotor impulse wheel, the condenser pressure being 0.5 atmosphere with the nozzle efficiency of 0.94 and the nozzle angle at the rotor inlet is 18^O to the wheel plane. The rotor blades which moves with the speed of 450 m/s are equiangular. If the coefficient of velocity for the rotor blades is 0.92, find (i) The specific power output (ii) The rotor efficiency (iii) The Stage efficiency (iv) Axial thrust (v) The direction of exit steam.

Solution : Given : p_1 = 10 atm ≈ 10 bar, p_o = 0.5 atm ≈ 0.5 bar, η_{nozzle} = 0.94, $\alpha_1 = 18^O$, U =450 m/s, $\beta_1 = \beta_2$, C_b= 0.92 = V_{r2} / V_{r1}.





From Mollier Chart : h_1= 2780 kJ/kg , h_2^1 = 2270 kJ/kg

Isentropic enthalpy change, $\Delta h^1 = h_1 - h^1_2 = 2780 - 2270$

$$\Delta h^1 = 510 \text{ kJ/kg}$$

Actual enthalpy change, $\Delta h = h_1 - h_2 = V_1^2 / 2g_c$ (as negligible velocity at nozzle inlet)

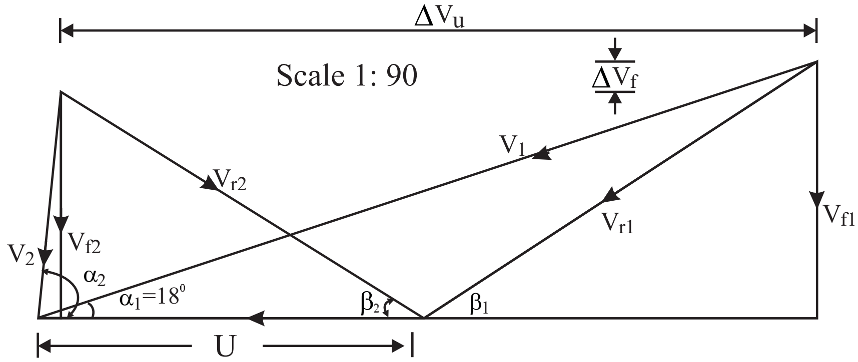
∴ Efficiency of the nozzle,

$$\eta_{nozzle} = \frac{\text{Actual enthalpy change /kg of steam nozzle}}{\text{Isentropic enthalpy change /kg of steam in nozzle}}$$

$$0.94 = (V_1^2 / 2g_c) / \Delta h^1$$

$$\therefore V_1 = 2 \times g_c \times 0.94 \times \Delta h^1 = 2 \times 1 \times 0.94 \times 510 \times 10^3$$

Velocity of steam entering the turbine is , V_1 = 979.2 m/s



From Graph : $\Delta V_f = 24.12$ m/s $\alpha_2 = 88.5^{\circ}$

$\Delta V_u = 924.1$ m/s

(i) **The specific power output** (P)

$$\text{Specific power output, } W = \frac{P}{\dot{m}} = \frac{U\,\Delta V_u}{g_c} = \frac{450 \times 924.1}{1000} = 415.9 \text{ kW/(kg/s)}$$

$$\text{(ii) \textbf{The rotor efficiency :} } \eta_b = \frac{W}{V_1^2/2g_c} = \frac{2U\,\Delta V_u}{V_1^2} = \frac{2 \times 450 \times 924.1}{(979.2)^2} \times 100$$

$$= 86.7\ \%$$

(iii) **Stage efficiency :** $\eta_{stage} = \eta_b \times \eta_{nozzle} = 0.867 \times 0.94$

$= 81.4\ \%$

(iv) **Axial thrust :** $F_a = \frac{\dot{m}}{g_c}\,\Delta V_f = \frac{1 \times 24.12}{1}$

$F_a = 24.12$ N / (kg/s)

(v) **The direction of exit steam** i.e.,

$\alpha_2 = 88.5^{\circ}$ in the direction of tangential speed of rotor.

6.0 Analysis on Two Stages :

6.1 Condition of Maximum Efficiency for Velocity Compounded Impulse Turbine (Curtis Turbine)

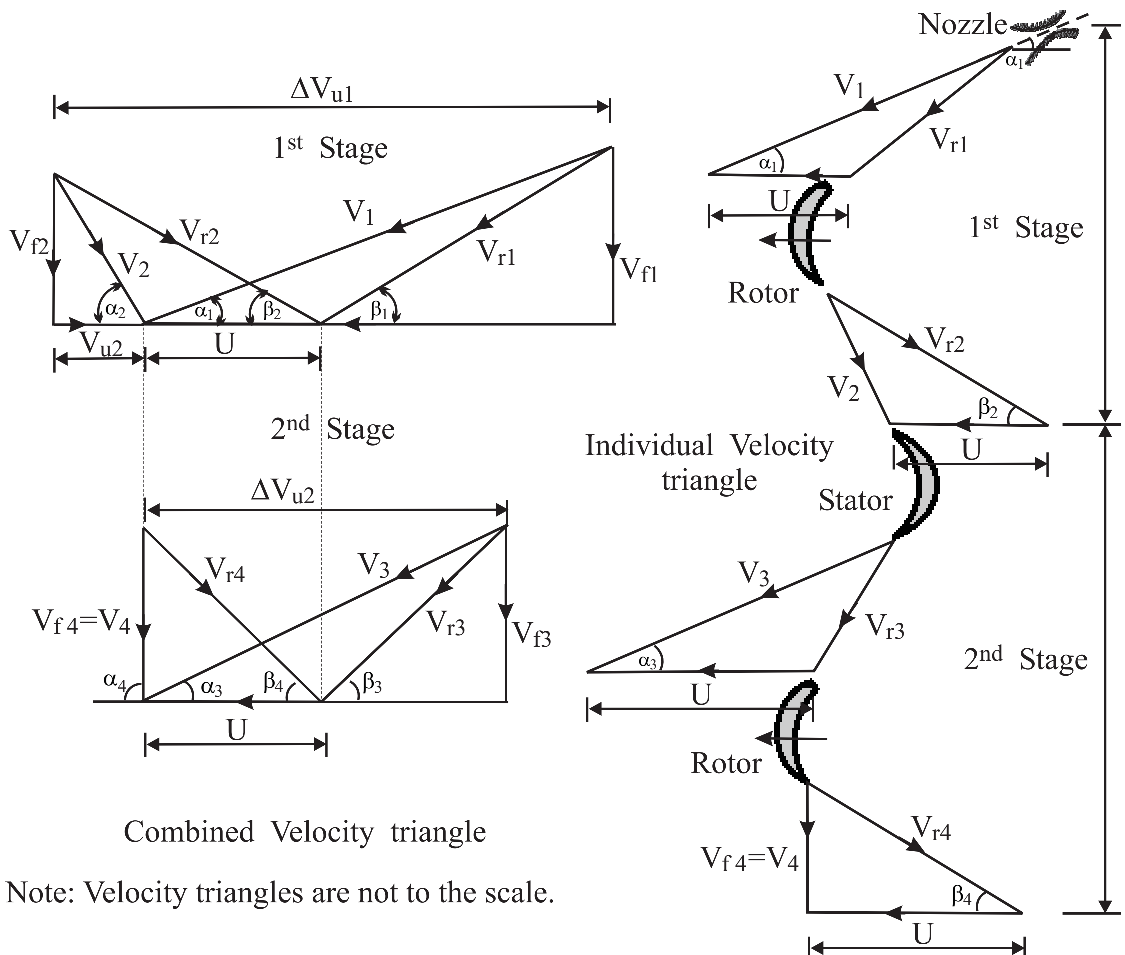


Fig. 10. Curtis two stage steam turbine.

The velocity triangles for first stage and second stage of a curtis turbine are as shown in Fig. 10. The speed and the angles should be so selected that the final absolute velocity of steam leaving the second row is axial, so as to obtain maximum efficiency. The tangential speed of blade for both the rows is same since all the moving blades are mounted on the same shaft.

In the first row of moving blades, the work done per kg of steam is,

$$W_1 = U(V_{u1}+V_{u2})/g_c = U\,\Delta V_{u1}/g_c$$

From I stage velocity triangle, we get,

$$W_1 = U(V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2)/g_c$$

If $\beta_1=\beta_2$ and $V_{r1}=V_{r2}$, then, $W_1 = 2U(V_{r1}\cos\beta_1)/g_c$

$$W_1 = 2U(V_1\cos\alpha_1 - U)/g_c \qquad (11)$$

The magnitude of the absolute velocity of steam leaving the first row is same as the velocity of steam entering the second row of moving blades, if there is no frictional loss (i.e., $V_2 = V_3$) but only the direction is going to be changed.

In the second moving row, work done per kg of steam is,

$$W_2 = U\,\Delta V_{u2}/g_c = U(V_{r3}\cos\beta_3 + V_{r4}\cos\beta_4)/g_c$$

Again if $\beta_3 = \beta_4$ and $V_{r3} = V_{r4}$

$$W_2 = 2U\,(V_{r3}\cos\beta_3)/g_c$$
$$= 2U\,(V_3\cos\alpha_3 - U)/g_c$$

If no loss in absolute velocity of steam entering from I rotor to the II rotor, then $V_3 = V_2$ & $\alpha_3 = \alpha_2$

$$\therefore\ W_2 = 2U\,(V_2\cos\alpha_2 - U)/g_c$$

Also, $V_2\cos\alpha_2 = V_{r2}\cos\beta_2 - U = V_{r1}\cos\beta_1 - U.$ $\quad(\because V_{r1} = V_{r2}\ \&\ \beta_1 = \beta_2$

$V_2\cos\alpha_2 = (V_1\cos\alpha_2 - U) - U$ $\quad(\because V_{r1}\cos\beta_1 = V_1\cos\alpha_1 - U)$

$$V_2\cos\alpha_2 = V_1\cos\alpha_1 - 2U$$

$$W_2 = 2U\,(V_1\cos\alpha_1 - 3U)/g_c \qquad (12)$$

Total Work done per kg of steam from both the stages is given by

$$W_T = W_1 + W_2 = 2U\,[V_1\cos\alpha_1 - 3U + V_1\cos\alpha_1 - U]/g_c$$
$$W_T = 2U\,[2V_1\cos\alpha_1 - 4U]/g_c$$
$$W_T = 4U\,[V_1\cos\alpha_1 - 2U]/g_c \qquad (13)$$

In general form, the above equation can be expressed as,

$$W_T = 2\,n\,U\,[V_1\cos\alpha_1 - nU]/g_c \qquad (14)$$

Where, n = number of stages.

Then the blade efficiency is given by,

$$\eta_b = \frac{W_T}{(V_1^2/2g_c)} = \frac{4U(V_1\cos\alpha_1 - 2U)}{V_1^2/2g_c} = \frac{8U}{V_1}\cos\alpha_1 - \frac{2U}{V_1} = 8\,\phi\,[\cos\alpha_1 - 2\phi] \qquad (15)$$

For maximum blade efficiency, $\dfrac{\partial\eta_b}{\partial\phi} = 0$

$$\partial\eta_b/\partial\phi = 8\,\phi\,[\cos\alpha_1 - 2\phi] \Rightarrow \cos\alpha_1 - 4\phi = 0$$

$$\therefore\quad \phi = \frac{\cos\alpha_1}{4} = U/V_1 \qquad (15\,a)$$

Then, the maximum blade efficiency is

$$\eta_{b,\max} = 8\,x\,\frac{\cos\alpha_1}{4}\quad \cos\alpha_1 - 2\,\frac{\cos\alpha_1}{4}$$

$$\eta_{b,\,max} = \cos^2\alpha_1 \qquad (15\,b)$$

Maximum total work done per kg of steam (from Eqn. 6.13) is,

$$W_{T,\,max} = 4\,U/g_c \frac{4U}{\cos\alpha_1} \cos\alpha_1 - 2U$$

$$W_{T,\,max} = 8\,U^2/g_c \qquad (16)$$

The present analysis is made for 2 stages only. The similar procedure can be adopted for analyzing the with three or more stages. In general, optimum blade speed ratio for maximum blade efficiency or maximum work done is given by

$$\phi = \frac{\cos\alpha_1}{2\,n} \qquad (17)$$

And work done in the last row $= \frac{1}{2^n}$ of total work (18)

As the number of moving rows increases (i.e., number of stages) the utility of last row decreases and hence we can achieve maximum utilization of all the kinetic energy available at the inlet which will be absorbed to the maximum extent among all moving rows. In practice more than two stages are adopted.

Note : i) Symbols used for velocity vector as follows :

V_1 = Absolute velocity of steam entering the first rotor or steam velocity at the exit of the nozzle, m/s.

α_1 = Nozzle angle with respect to wheel plane or tangential blade speed at inlet to the first rotor, degrees.

V_{r1} = Relative velocity of fluid at inlet of I stage, m/s.

β_1 = Rotor or blade angle at inlet of 1st rotor made by V_{r1}, degree

V_{r2} = Relative velocity of fluid at outlet of I stage.= ($C_{b1} V_{r1}$)

β_2 = Rotor or blade angle at outlet of I rotor made by V_{r2}, degree

V_2 = Absolute velocity of steam leaving the I rotor or stage, m/s

α_2 = Exit angle of steam made by V_2, degree

and V_3 = Absolute velocity of steam entering the 2nd rotor or exit velocity of the steam from the stator = ($C_{b2} V_2$) m/s.

α_3 = Exit angle of stator for 2nd rotor, degree

V_{r3}, β_3, V_{r4} (= $C_{b3} V_{r3}$), β_4, V_4, are corresponding values in the second stage.

(ii) If the fluid discharges axially at the last row means, $V_4 = V_{f4}$ i.e., $\alpha_4 = 90^\circ$, $V_{u4} = 0$.

(iii) If the tangential speed or velocity of blade to be found when the axial discharge is given, then one should start always from the II stage by assuming suitable length for U and then proceed to the I stage. Finally, by finding the actual scale ratio with the known quantity (say V_1 is given) find out the value of U and other parameters by using scale ratio obtained from graph.

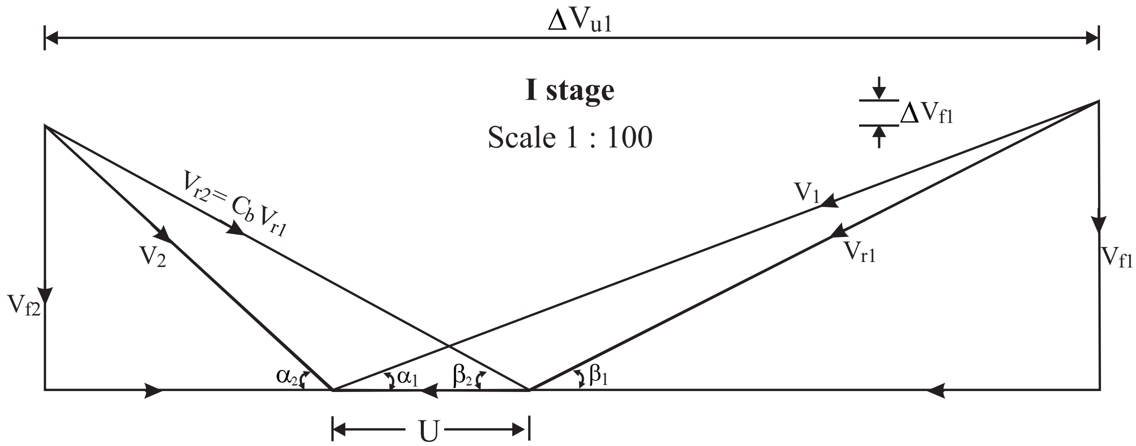
(iv) Especially for two-stage turbine, graphical method is preferable, otherwise more calculation procedure is involved in analytical method.

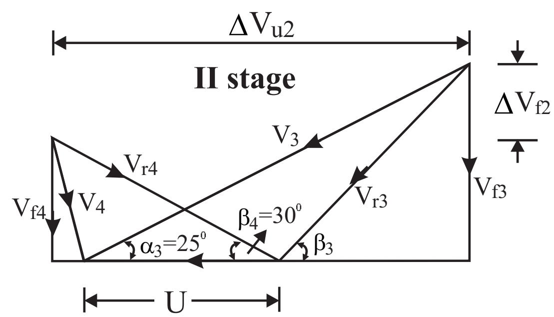
Example 4: The following data refers to a velocity compounded Impulse steam turbine having two rows of moving blades and a fixed row between them : Velocity of steam leaving the nozzle is 1200 m/s, nozzle angle is 20^0, blade speed is 250 m/s, blade angles of first moving row are equiangular, blade outlet angle of the fixed blade is 25^O. Blade outlet angle of the second moving row is 30^O. Friction factor for all the rows is 0.9. Draw the velocity diagrams for a suitable scale and calculate the power developed, axial thrust, diagram efficiency for steam flow rate of 5000 kg/hr.

Solution :Given : $V_1 = 1200$ m/s, $\alpha_1 = 20^O$, $U = 250$ m/s, $\beta_1 = \beta_2$, $\alpha_3 = 25^O$, $\beta_4 = 30^O$,

$C_b = V_{r2}/V_{r1} = V_3/V_2 = V_{r4}/V_{r3} = 0.9$, $\dot{m} = 1.389$ kg/s. *To find*: P, F_a, η_b.

Velocity triangles are constructed by graphical method assuming a scale of 1:100





From Graph : $\Delta V_{u1} = 1667.5$ m/s, $\Delta V_{f1} = 41.1$ m/s

$\Delta V_{u2} = 577.4$ m/s, $\Delta V_{f2} = 79$ m/s

(i) Power developed : $P = \dot{m}\ U[\Delta V_{u1} + \Delta V_{u2}]/g_c = \dfrac{1.389 \times 250\,(1667.5 + 577.4)}{1000}$

$P = 779.54$ kW

(ii) Axial thrust, (F_a)

$$F_a = \dot{m}\,[\Delta V_{f1}+\Delta V_{f2}]/g_c = \frac{1.389\,(41.1+79)}{1}$$

$$F_a = 166.8\ \text{N}$$

(iii) Diagram efficiency (η_b) :

$$\eta_b = \frac{2U(\Delta V_{u1}+\Delta V_{u2})}{V_1^2} = \frac{2 \times 250\,[1666.7+577.4]}{(1200)^2} \times 100\,\%$$

$$\eta_b = 77.95\ \%$$

Example 5 : The velocity of steam at the exit of a nozzle is 440 m/s which is compounded in an Impulse turbine by passing successively through moving, fixed, and finally through a second ring of moving blades. The tip angles of moving blades throughout the turbine are 30°. Assume loss of 10% in velocity due to friction when the steam passes over a blade ring. Find the velocity of moving blades in order to have a final discharge of steam as axial. Also determine the diagram efficiency.

Solution : Given : $V_1 = 440$ m/s, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 30^{\circ}$, $C_b = 0.9 = \frac{V_{r2}}{V_{r1}} = \frac{V_3}{V_2} = \frac{V_{r4}}{V_{r3}}$

To find : U & η_b.

Note : As we have to find out the tangential speed of rotors for the axial discharge at the last row, we have to proceed from the 2^{nd} stage by assuming a suitable length for U. We assumed U = 3 cm, Finally from Graph we get, $V_1 = 13.6$ cm .

Also, $V_1 = 440$ m/s = 13.6 cm ($\because$ $V_1 = 440$ m/s given)

$\therefore$ Scale ratio = 1 : 32.35 i.e., 1 cm = 32.35 m/s

(i) Tangential velocity of rotor (U)

$\therefore$ U = 3 cm x 32.35 = 97.06 m/s

(ii) Diagram efficiency (η_b) :

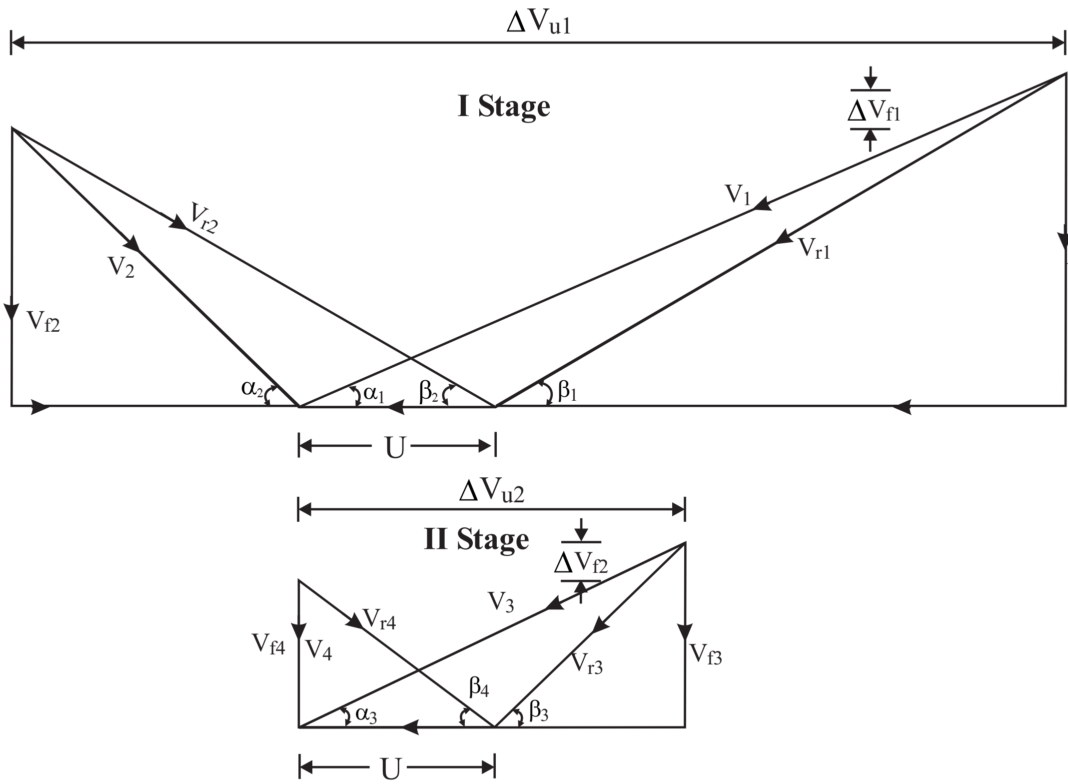
$$\eta_b = \frac{2U(\Delta V_{u1}+\Delta V_{u2})}{V_1^2}$$

From Graph :

$\Delta V_{u1} = 579$ m/s

$\Delta V_{u2} = 204.8$ m/s

$$\eta_b = \frac{2 \times 97.06\,[579 + 204.8]}{(440)^2} = 78.6\,\%$$



7.0 Reaction Turbines :

The reaction turbine using now-a-days are of Impulse-Reaction turbines. Pure reaction turbines are not in general use. The expansion of steam and heat drop occur both in fixed and moving blades. All reaction turbines are of axial flow type.

7.1 Velocity Triangles for General case :

Velocity diagrams are shown in Fig.11. In reaction turbine, steam continuously expands as it flows over the blades. The effect of the continuous expansion during the flow over the moving blades is to increase the relative velocity of steam ie., $V_{r2} > V_{r1}$,

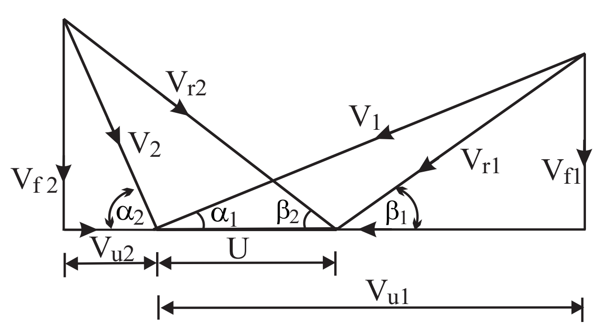


Fig.11.Impulse Reaction turbine stages.

7.2 Degree of Reaction (R) :

The degree of reaction for reaction turbine stage is defined as *the ratio of enthalpy drop in the moving blades to the total enthalpy drop in fixed and moving blades (i.e., static enthalpy drop to total enthalpy drop),* as shown in Fig. 12.

Thus ,

$$R = \frac{\text{Enthalpy drop in moving blades}}{\text{Total enthalpy drop in a stage}}$$

$$= \frac{\Delta h_m}{\Delta h_f + \Delta h_m}$$

Enthalpy drop in moving blades

$$\frac{V_{r2}^2 - V_{r1}^2}{2g_c}$$

Total enthalpy drop in a stage :

$$\Delta h_0 = W = \Delta h_f + \Delta h_m = \frac{U}{g_c}(V_{u1} + V_{u2})$$

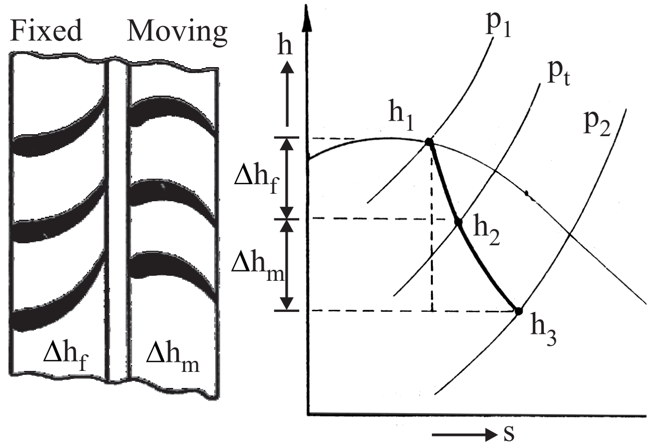


Fig.12 h-s. diagram for enthalpy drop.

$$\therefore \quad R = \frac{(V_{r2}^2 - V_{r1}^2)/2g_c}{U(V_{u1} + V_{u2})/g_c} = \frac{V_{r2}^2 - V_{r1}^2}{2U(V_{u1} + V_{u2})}$$

From velocity triangle : $V_{r2} = V_{f2} \operatorname{cosec} \beta_2$ & $V_{r1} = V_{f1} \operatorname{cosec} \beta_1$

and $(V_{u1} + V_{u2}) = V_{f1} \cot \beta_1 + V_{f2} \cot \beta_2$

$V_{f1} = V_{f2} = V_f$ (because flow velocity is constant generally)

$$\therefore \quad R = \frac{V_f^2}{2U} \frac{\operatorname{cosec}^2 \beta_2 - \operatorname{cosec}^2 \beta_1}{\cot \beta_1 + \cot \beta_2} = \frac{V_f}{2U} \frac{(1 + \cot^2 \beta_2) - (1 + \cot^2 \beta_1)}{\cot \beta_1 + \cot \beta_2}$$

$$R = \frac{V_f}{2U} \frac{\cot^2 \beta_2 - \cot^2 \beta_1}{\cot \beta_1 + \cot \beta_2}$$

$$R = \frac{V_f}{2U} [\cot \beta_2 - \cot \beta_1] \qquad (19)$$

If $R = 0.5$ i.e., 50% Reaction, then

$$U = V_f [\cot \beta_2 - \cot \beta_1] \qquad (20)$$

7.3 Analysis on 50% Reaction (Parson's) Turbine :

When $R = 0.5$, then $V_1 = V_{r2}$ and $V_2 = V_{r1}$, also $\beta_2 = \alpha_1$ & $\beta_1 = \alpha_2$

Then the velocity Δ^{le} become symmetric as shown in Fig. 13.

Condition for Maximum Efficiency :

For maximum efficiency derivation, the following are assumed,

(i) Reaction is 50%

(ii) The moving and fixed blades are symmetrical

(iii) The velocity of steam leaving the first stage is same as the velocity of steam entering the next stage (i.e., there is no loss, $V_2 = V_3$)

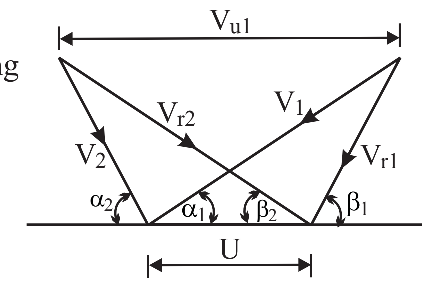


Fig.13. Parson's Reaction Turbine.

Work done per kg of Steam,

$$W = \frac{U \Delta V_u}{g_c} = \frac{U}{g_c} \left[V_1 \cos\alpha_1 + V_2 \cos\alpha_2 \right]$$

$$W = \frac{U}{g_c} \left[V_1 \cos\alpha_1 + (V_{r2} \cos\beta_2 - U) \right]$$

but $V_{r2} = V_1$ and $\beta_2 = \alpha_1$ for $R = 0.5$

$$W = U(2V_1 \cos\alpha_1 - U)/g_c = (2UV_1 \cos\alpha_1 - U^2)/g_c$$

In terms of speed ratio, $\phi = U/V_1$

$$W = \frac{V_1^2}{g_c} \left[2\phi \cos\alpha_1 - \phi^2 \right] \qquad (21)$$

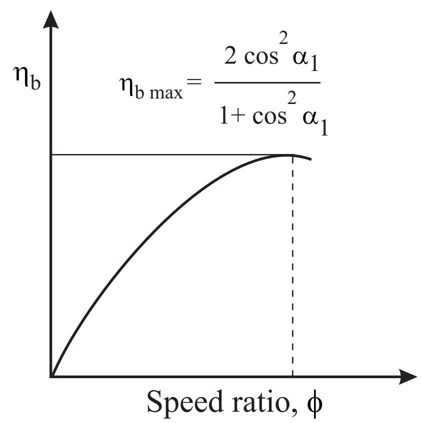


Fig.14. Speed ratio v/s blade efficiency.

The K.E. supplied to the fixed blade $= \dfrac{V_1^2}{2g_c}$

& K.E. supplied to the moving blade $= \dfrac{V_{r2}^2 - V_{r1}^2}{2g_c}$

Total energy supplied to the stage, $\Delta h_0 = \dfrac{V_1^2}{2g_c} + \dfrac{V_{r2}^2 - V_{r1}^2}{2g_c}$

$$\Delta h_0 = \frac{V_1^2}{2g_c} + \frac{V_{r2}^2}{2g_c} - \frac{V_{r1}^2}{2g_c}$$

$$\Delta h_0 = \frac{V_1^2}{g_c} - \frac{V_{r1}^2}{2g_c} \qquad (\because \; V_1 = V_{r2})$$

From inlet velocity triangle : $V_{r1}^2 = V_1^2 + U^2 - 2V_1 U \cos\alpha_1$

$$\therefore \qquad \Delta h_0 = \frac{V_1^2}{g_c} - \frac{(V_1^2 + U^2 - 2V_1 U \cos\alpha_1)}{2g_c}$$

$$\Delta h_0 = (V_1^2 + 2V_1 U \cos\alpha_1 - U^2)/2g_c$$

$$\Delta h_0 = \frac{V_1^2}{2g_c} \left[1 + \frac{2U}{V_1} \cos\alpha_1 - \left(\frac{U}{V_1} \right)^2 \right] = \frac{V_1^2}{2g_c} \left[1 + 2\phi \cos\alpha_1 - \phi^2 \right] \qquad (22)$$

The blade efficiency of reaction turbine is defined as,

$$\eta_b = \frac{\text{Work done per kg of steam}}{\text{Total energy supplied to stage}} = \frac{W}{\Delta h_0}$$

$$\eta_b = \frac{V_1^2\ [2\phi\cos\alpha_1 - \phi^2]/g_c}{V_1^2/2g_c[1+2\phi\ \cos\alpha_1 - \phi^2]} = \frac{2(2\phi\cos\alpha_1 - \phi^2)}{[1+2\phi\cos\alpha_1 - \phi^2]}$$

$$\eta_b = \frac{2(1+2\phi\cos\alpha_1 - \phi^2) - 2}{1+2\phi\cos\alpha_1 - \phi^2}$$

$$\eta_b = 2 - \frac{2}{1+2\phi\cos\alpha_1 - \phi^2} \qquad (23)$$

For maximum blade efficiency, $\frac{\partial \eta_b}{\partial \phi} = 0$

$$\therefore \frac{\partial}{\partial\phi}(1+2\phi\cos\alpha_1 - \phi^2) = 2\cos\alpha_1 - 2\phi = 0$$

$$\text{Speed ratio:}\quad \phi = \frac{U}{V_1} = \cos\alpha_1 \qquad (24)$$

$\therefore$ Equation for maximum blade efficiency,

$$\eta_{b,\max} = 2 - \frac{2}{1+2(\cos\alpha_1\ \cos\alpha_1) - \cos^2\alpha_1} = \frac{2(1+\cos^2\alpha_1) - 2}{1+\cos^2\alpha_1}$$

$$\eta_{b,\max} = \frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1} \qquad (25)$$

The variation of η_b with blade speed ratio ϕ for the reaction stage is shown in Fig. 14.

8.0 Effect of Reheat Factor and Stage Efficiency:

The thermodynamic effect on the turbine efficiency can be best understood by considering a number of stages, say 4, between states 1 & 5 as shown in Fig.15. Total expansion is divided into four stages of the same stage efficiency and pressure ratio.

i.e., $\frac{p_1}{p_2} = \frac{p_2}{p_3} = \frac{p_3}{p_4} = \frac{p_4}{p_4}$

Let η_0 is the overall efficiency of expansion and is defined as *the ratio of actual work done per kg of steam to the isentropic work done per kg of steam between 1 & 5.*

i.e., $\eta_o = \frac{W_a}{W_s}$ $\qquad \because \eta_o = \frac{h_1 - h_5}{h_1 - h_5^1}$

or the actual work done per kg of steam,

$$W_a = \eta_o W_s \qquad (26)$$

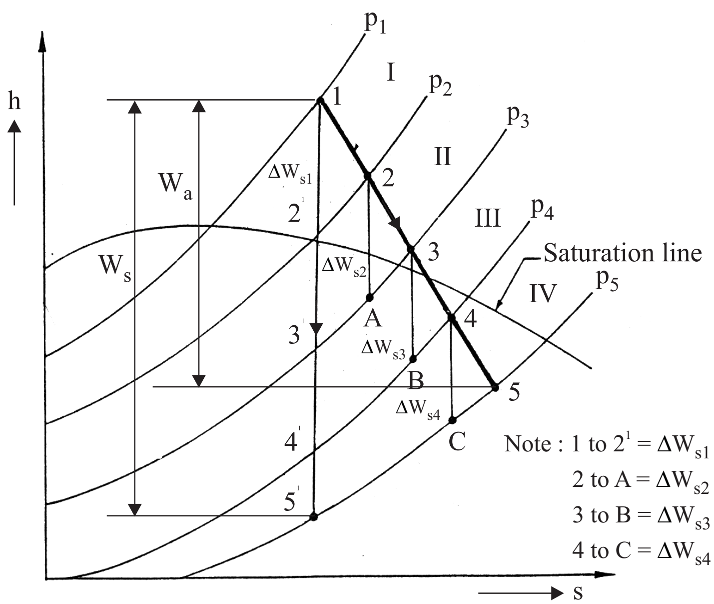


Fig.15. Enthalpy-entropy diagram for multi stage expansion.

Isentropic or ideal values in each stages are ΔW_{s1}, ΔW_{s2}, ΔW_{s3}, ΔW_{s4}.
Therefore, the total value of the actual work done in these stages is,

$$W_a = \Sigma \Delta W_a = \Sigma\,(1-2)+(2-3)+(3-4)+(4-5)$$

Also stage efficiency for each stage is given

$$\eta_s = \frac{\text{actual work done / kg of steam}}{\text{Isentropic work in a stage}} = \frac{W_{a1}}{W_{s1}}$$

For stage 1, $\eta_{s1} = \dfrac{W_{a1}}{W_{s1}} = \dfrac{h_1 - h_2}{h_1 - h'_2} = \dfrac{W_{a1}}{\Delta W_{s1}}$ or $\Delta W_{a1} = \eta_{s1}\,\Delta W_{s1}$

$\therefore$ $W_a = \Sigma \Delta W_a = \Sigma\,[\eta_{s1}\Delta W_{s1} + \eta_{s2}\Delta W_{s2} + \eta_{s3}\,\Delta W_{s3} + \eta_{s4}\,\Delta W_{s4}]$

For same stage efficiency in each stage, $\eta_{s1} = \eta_{s2} = \eta_{s3} = \eta_{s4}$

$$W_a = \eta_s \Sigma\,[\Delta W_{s1} + \Delta W_{s2} + \Delta W_{s3} + \Delta W_{s4}] = \eta_s\,\Sigma\,\Delta W_s \qquad (27)$$

From equation (26) and (27),

$$\eta_o\,W_s = \eta_s\,\Sigma\,\Delta W_s$$

$$\therefore \qquad \eta_o = \eta_s\,\frac{\Sigma \Delta W_s}{W_s} \qquad (28)$$

The slope of constant pressure lines on h-s plane is given by

$$\left(\frac{\partial h}{\partial s}\right)_p = T$$

This shows that the constant pressure lines must diverge towards the right. Therefore,

$$\frac{\sum \Delta W_s}{W_s} > 1,$$

for expansion process. It is obvious that the enthalpy increases when we move towards right along the constant pressure line. Hence the summation of ΔW_{s1}, ΔW_{s2} etc., is more than the total isentropic enthalpy drop, W_s

The ratio of summation of isentropic enthalpy drop for individual stage to the total isentropic enthalpy drop as a whole is called *Reheat factor*. Thus,

$$RF = \frac{\Sigma[\Delta W_{s1}+\Delta W_{s2}+\Delta W_{s3}+\Delta W_{s4}]}{W_s} = \frac{\Sigma\,[1-2^{1})+(2\ +a^{1})+(3-b^{1})+(4-c^{1}]}{(1-5)}$$

$$RF = \frac{\Sigma \Delta W_s}{W_s} \qquad (29)$$

$\therefore$ The overall efficiency of expansion precess, $\eta_O = \eta_{stage} \text{ x } RF$ (30)

As $RF = (\sum \Delta W_s / W_s) > 1$, the overall efficiency of turbine (η_O) is greater than the stage efficiency η_s.

i.e., $\eta_o > \eta_s$ for turbines. (31)

8.1 Advantages of Reheating :

(1) There is an increase output of turbine

(2) Erosion and corrosion problems in the steam turbine are eliminated or avoided

(3) There is an improvement in the thermal efficiency of the turbines.

(4) Final dryness fraction of steam is improved.

(5) There is an increase in blade and nozzle efficiencies.

8.2 Demerits :

(1) Reheating requires more maintenance.

(2) The increase in thermal efficiency is not appreciable compared to expenditure incurred in reheating.

Example 6.: The following data refers to a particular stage of a Parson's reaction turbine. Speed of the turbine = 1500 RPM, Mean diameter of the rotor = 1m, stage efficiency = 0.8, Blade outlet angle = 20^{O}, Speed ratio = 0.7, Determine the available isentropic enthalpy drop in the stage.

Solution : $N = 1500$ RPM, $D = 1$m, $\eta_s = 0.8$, $\beta_2 = \alpha_1 = 20^{O}$, $\phi = U/V_1 = 0.7$, *find* : Δh^{1}

Tangential speed of rotor, $U = \frac{\pi DN}{60} = \frac{\pi \text{ x } 1 \text{ x } 1500}{60} = 78.54$ m/s

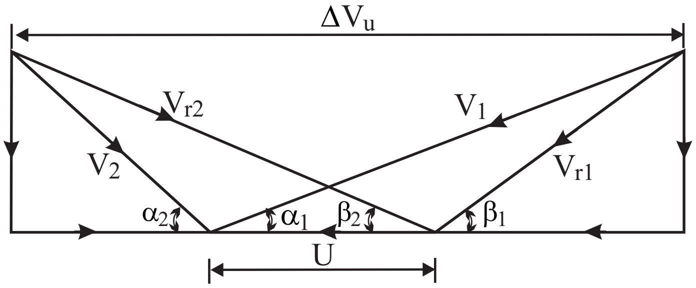
$\therefore$ Steam velocity at inlet of stage, $V_1 = \frac{U}{\phi} = \frac{78.54}{0.7} = 112.2$ m/s

Work done per kg of steam,

$$W = \frac{U}{g_c}(\Delta V_u)$$

From velocity triangle :

$$\Delta V_u = V_1 \cos\alpha_1 + (V_{r2}\cos\beta_2 - U)$$
$$= V_1 \cos\alpha_1 + V_1 \cos\alpha_1 - U$$
$$= 2V_1 \cos\alpha_1 - U \quad (\because \alpha_1 = \beta_2 \text{ as } V_{r2} = V_1)$$
$$\Delta V_u = 2 \times 112.2 \times \cos 20^0 - 78.54 = 132.33 \text{ m/s}$$



$$\therefore \quad W = \frac{78.54 \times 132.33}{1 \times 1000} = 10.39 \text{ kJ/kg}$$

Stage efficiency is given by,

$$\eta_s = \frac{\text{Work done per kg of steam}}{\text{Isentropic enthalpy drop in a stage}} = \frac{W}{\Delta h^1}$$

$$\therefore \quad \text{Isentropic enthalpy drop, } \Delta h^1 = \frac{W}{\eta_s} = \frac{10.39}{0.8}$$
$$\Delta h^1 = 13 \text{ kJ/kg}$$

Example 7: In a reaction turbine, the blade tips are inclined at 35° and 20° in the direction of rotor. The stator blades are the same shape as the moving blades, but reversed in direction. At a certain place in the turbine, the drum is 1m diameter and the blades are 10 cm high. At this place, the steam has a pressure of 1.75 bar and dryness is 0.935. If the speed of the turbine is 250 RPM and the steam passes through the blades without shock find the mass of steam flow and power developed in the ring of moving blades.

Solution : Given : $\beta_1 = 35^\circ = \alpha_2$, $\beta_2 = 20^\circ = \alpha_1$, D= 1m, N = 250 RPM, h = 0.1 m, At certain point, p = 1.75 bar, X = 0.935. *Find* : $\dot{m}$, P.

Tangential speed of rotor

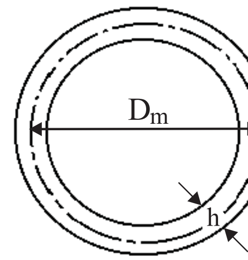
$$U = \frac{\pi D_m N}{60}$$

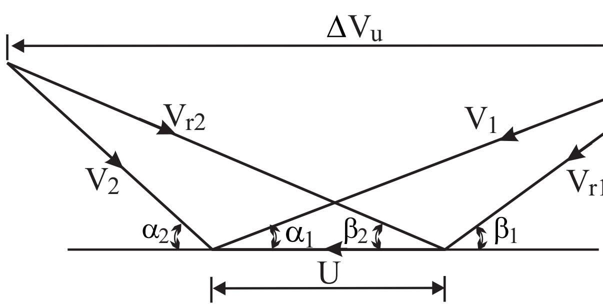
Where D_m = Mean diameter of rotor = D + h

$$D_m = 1.0 + 0.1 = 1.1 \text{m}$$

$$U = \frac{\pi \times 1.1 \times 250}{60} = 14.4 \text{ m/s}$$

Area of flow, $A_f = \pi D_m h = \pi \times 1.1 \times 0.1 = 0.3456 \text{ m}^2$





From Graph : $\Delta V_u = 45.45$ m/s

$V_{f1} = V_{f2} = 10.8 \text{ m/s} = V_f$

From Mollier chart,

Specific volume of steam, v = 0.96 m^3/kg corresponding to pressure = 1.75 bar and dryness fraction, X= 0.935.

$\therefore$ Density of steam at that point, $\rho = \frac{1}{v} = 1.042 \text{ kg/m}^3$

(a) Mass flow rate ($\dot{m}$):

$$\dot{m} = \rho A_f V_f = 1.042 \times 0.3456 \times 10.8 \text{ kg/s}$$

$$\dot{m} = 3.89 \text{ kg/s}$$

(b) Power developed (P) :

$$P = \frac{\dot{m}}{g_c} U \Delta V_u = \frac{3.89 \times 14.4 \times 45.45}{1000} \text{ kW}$$

$$P = 2.545 \text{ kW}$$