

MATHEMATICS

Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that

$P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals

- (A) 52 (B) 103 (C) 201 (D) 205

Let $b_i > 1$ for $i = 1, 2, \dots, 101$. Suppose $\log_e b_1, \log_e b_2, \dots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose a_1, a_2, \dots, a_{101} are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \dots + b_{51}$ and $s = a_1 + a_2 + \dots + a_{51}$, then

- (A) $s > t$ and $a_{101} > b_{101}$ (B) $s > t$ and $a_{101} < b_{101}$
 (C) $s < t$ and $a_{101} > b_{101}$ (D) $s < t$ and $a_{101} < b_{101}$

The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to

- (A) $3 - \sqrt{3}$ (B) $2(3 - \sqrt{3})$ (C) $2(\sqrt{3} - 1)$ (D) $2(2 + \sqrt{3})$

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1 + e^x} dx$ is equal to

- (A) $\frac{\pi^2}{4} - 2$ (B) $\frac{\pi^2}{4} + 2$ (C) $\pi^2 - e^{\frac{\pi}{2}}$ (D) $\pi^2 + e^{\frac{\pi}{2}}$

Area of the region $\{(x, y) \in \mathbb{R}^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\}$ is equal to

- (A) $\frac{1}{6}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$

Let P be the image of the point $(3, 1, 7)$ with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

- (A) $x + y - 3z = 0$ (B) $3x + z = 0$
 (C) $x - 4y + 7z = 0$ (D) $2x - y = 0$

Let $f: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ and $g: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

- (A) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
- (B) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
- (C) g is **NOT** differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$
- (D) g is **NOT** differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Suppose $S = \left\{z \in \mathbb{C} : z = \frac{1}{a + ibt}, t \in \mathbb{R}, t \neq 0\right\}$, where $i = \sqrt{-1}$.

If $z = x + iy$ and $z \in S$, then (x, y) lies on

- (A) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$
- (B) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$
- (C) the x -axis for $a \neq 0, b = 0$
- (D) the y -axis for $a = 0, b \neq 0$

Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then

- (A) $SP = 2\sqrt{5}$
- (B) $SQ : QP = (\sqrt{5} + 1) : 2$
- (C) the x -intercept of the normal to the parabola at P is 6
- (D) the slope of the tangent to the circle at Q is $\frac{1}{2}$

Let $\alpha, \lambda, \mu \in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement(s) is(are) correct?

- (A) If $\alpha = -3$, then the system has infinitely many solutions for all values of λ and μ
- (B) If $\alpha \neq -3$, then the system has a unique solution for all values of λ and μ
- (C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $\alpha = -3$
- (D) If $\lambda + \mu \neq 0$, then the system has no solution for $\alpha = -3$

Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in \mathbb{R}^3 and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector \vec{v} in \mathbb{R}^3 such that $|\hat{u} \times \vec{v}| = 1$ and $\hat{w} \cdot (\hat{u} \times \vec{v}) = 1$. Which of the following statement(s) is(are) correct?

- (A) There is exactly one choice for such \vec{v}
- (B) There are infinitely many choices for such \vec{v}
- (C) If \hat{u} lies in the xy -plane then $|u_1| = |u_2|$
- (D) If \hat{u} lies in the xz -plane then $2|u_1| = |u_3|$

PARAGRAPH 1

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 , respectively, after two games.

$P(X > Y)$ is

- (A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{12}$

PARAGRAPH 2

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

The orthocentre of the triangle F_1MN is

- (A) $\left(-\frac{9}{10}, 0\right)$ (B) $\left(\frac{2}{3}, 0\right)$ (C) $\left(\frac{9}{10}, 0\right)$ (D) $\left(\frac{2}{3}, \sqrt{6}\right)$

PARAGRAPH 2

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x -axis at Q , then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

- (A) 3 : 4 (B) 4 : 5 (C) 5 : 8 (D) 2 : 3